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GRADE 11 MATHS

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CONTENT

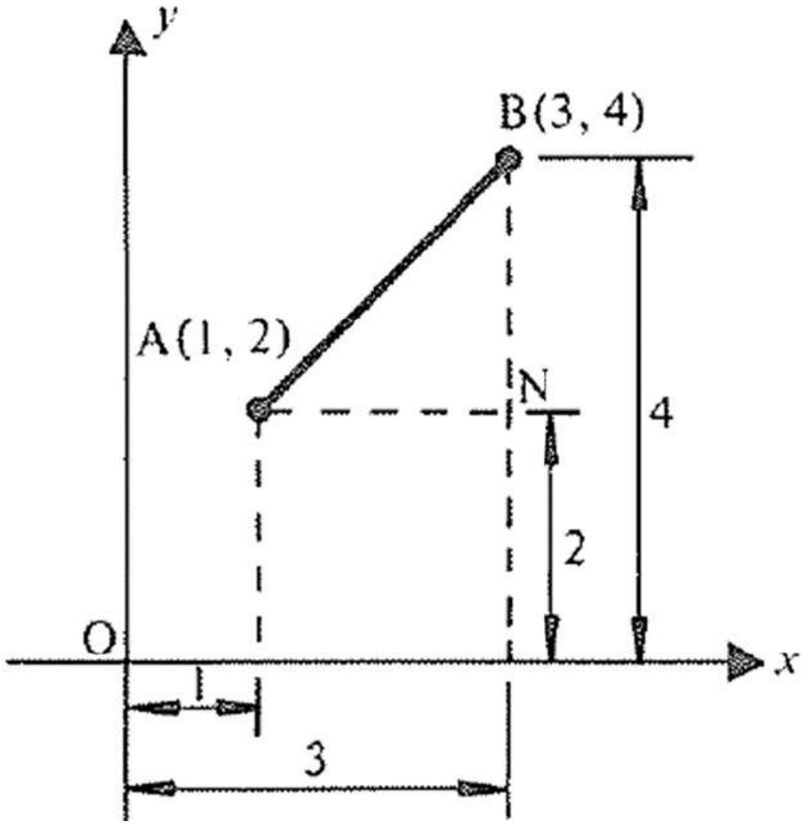
- Equation of a line
- Inclination of a line
- Parallel lines
- Perpendicular lines
- Geometry theorems
- Equation of a tangent to the circle

LESSON OBJECTIVES

NOTES

- that parallel lines have equal gradients and equal angles of inclination.
- that the product of the gradients of perpendicular lines is equal to -1.
- that horizontal lines have a zero gradient.
- that vertical lines have an undefined gradient

LENGTH OF A LINE



From the diagram we see that the length of the line joining A(1, 2) and B(3, 4) can be found using Pythagoras' Theorem where

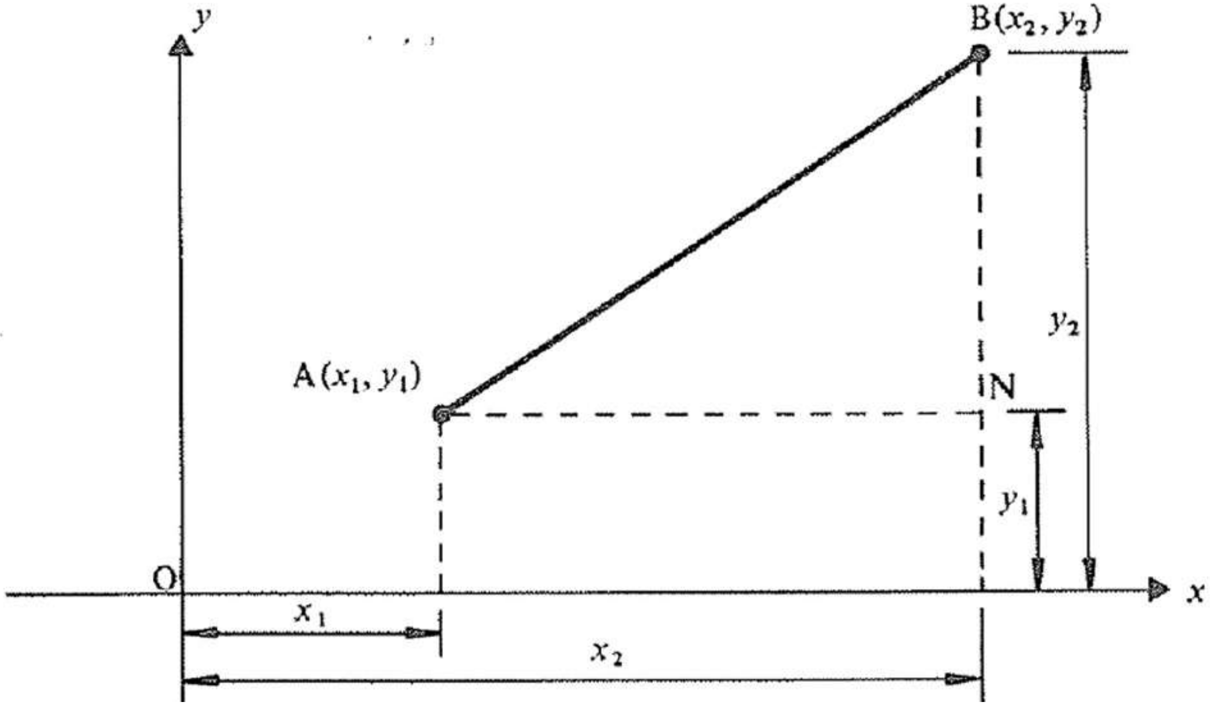
$$AB^2 = AN^2 + BN^2$$

$$= (3-1)^2 + (4-2)^2$$

$$= 8$$

Therefore $AB = \sqrt{8} = 2\sqrt{2}$.

In general, if $A(x_1, y_1)$ and $B(x_2, y_2)$ are any two points



we see from the diagram, and using Pythagoras' Theorem, that

$$AB^2 = AN^2 + BN^2$$

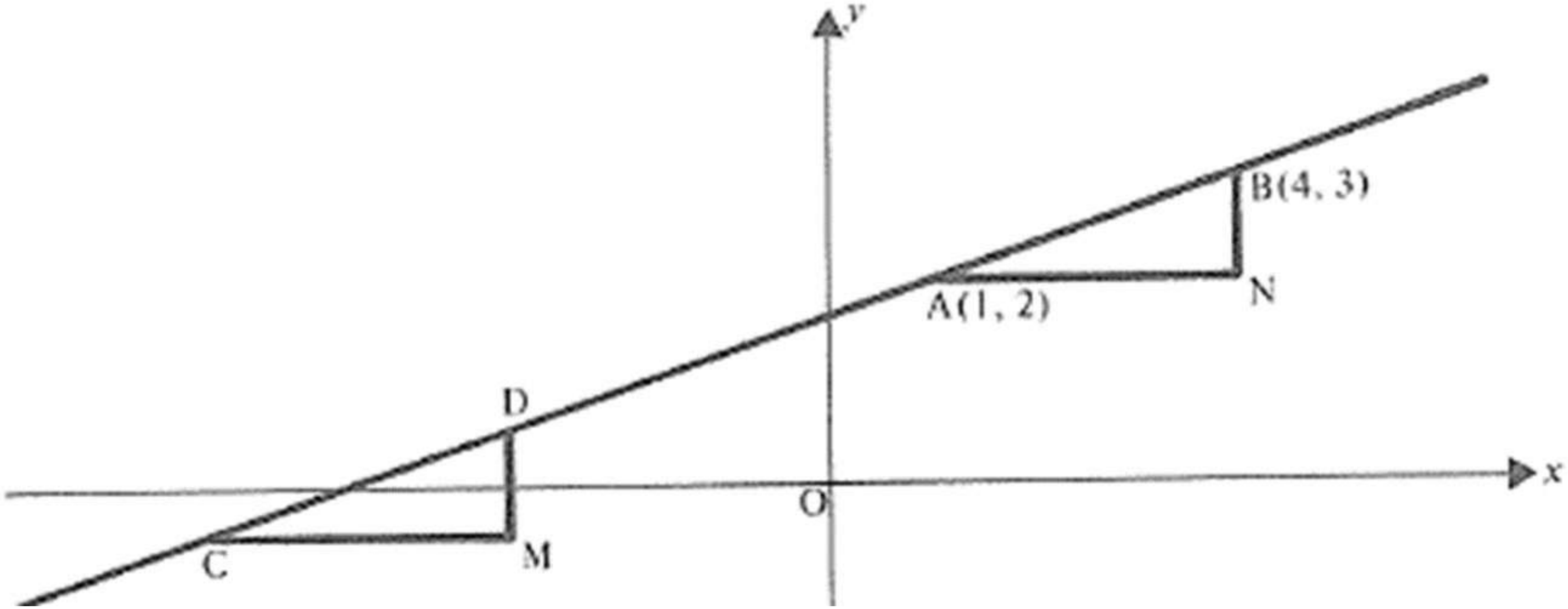
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Therefore $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note that this formula still holds when some, or all, of the coordinates are negative, e.g.

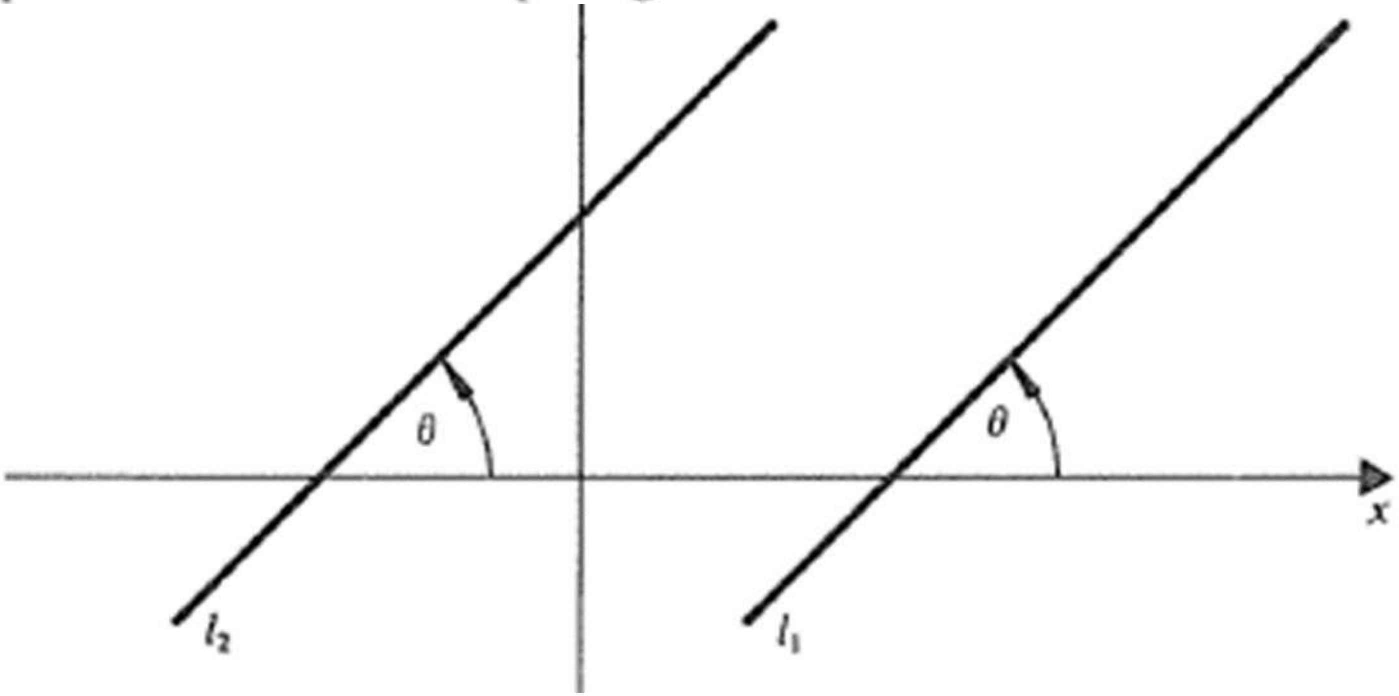
GRADIENT

- The gradient of a straight line is a measure of its slope with respect to the x-axis, and is defined as the increase in the y coordinate divided by the increase in the x coordinate between one point on the line and another point on the line.

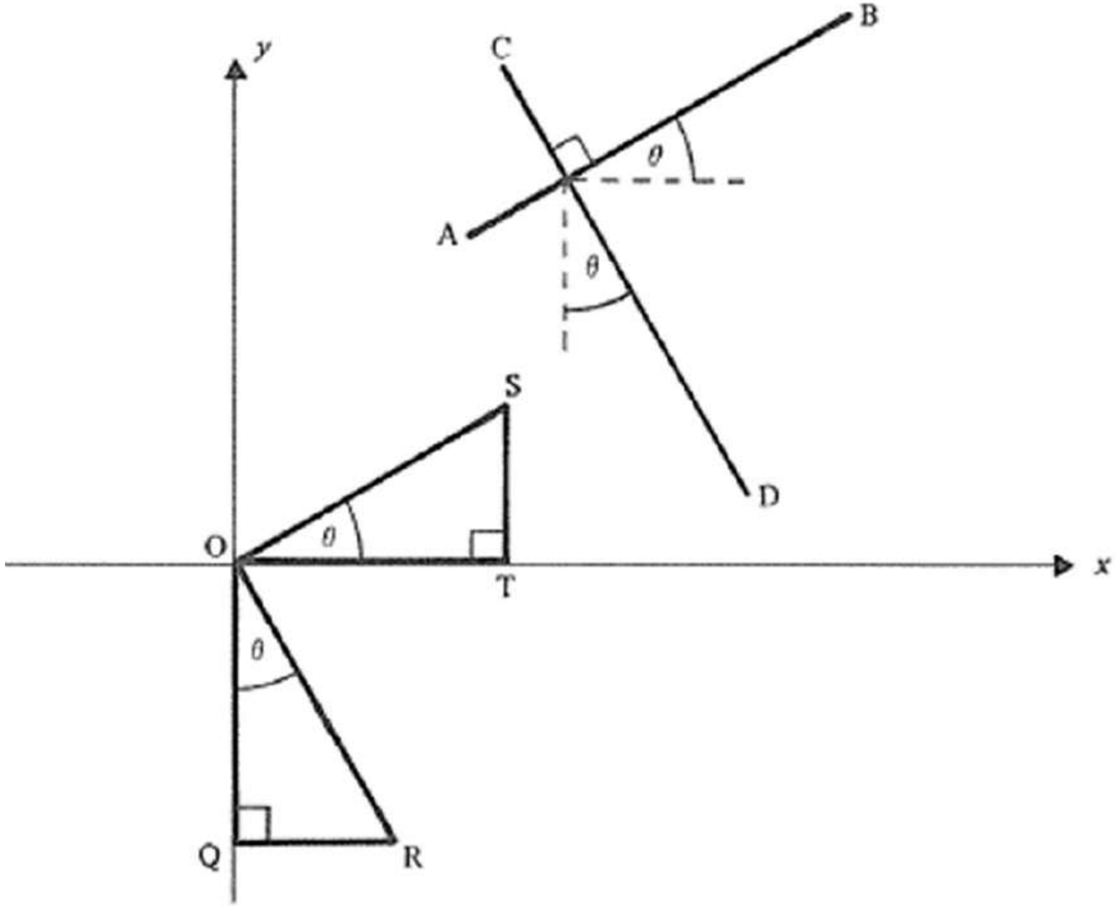


PARALLEL LINES

If l_1 and l_2 are parallel lines, they are equally inclined to the positive direction of the x-axis (corresponding angles), so $\tan\theta$ is the gradient of both l_1 and l_2, i.e. *parallel lines have equal gradients.*

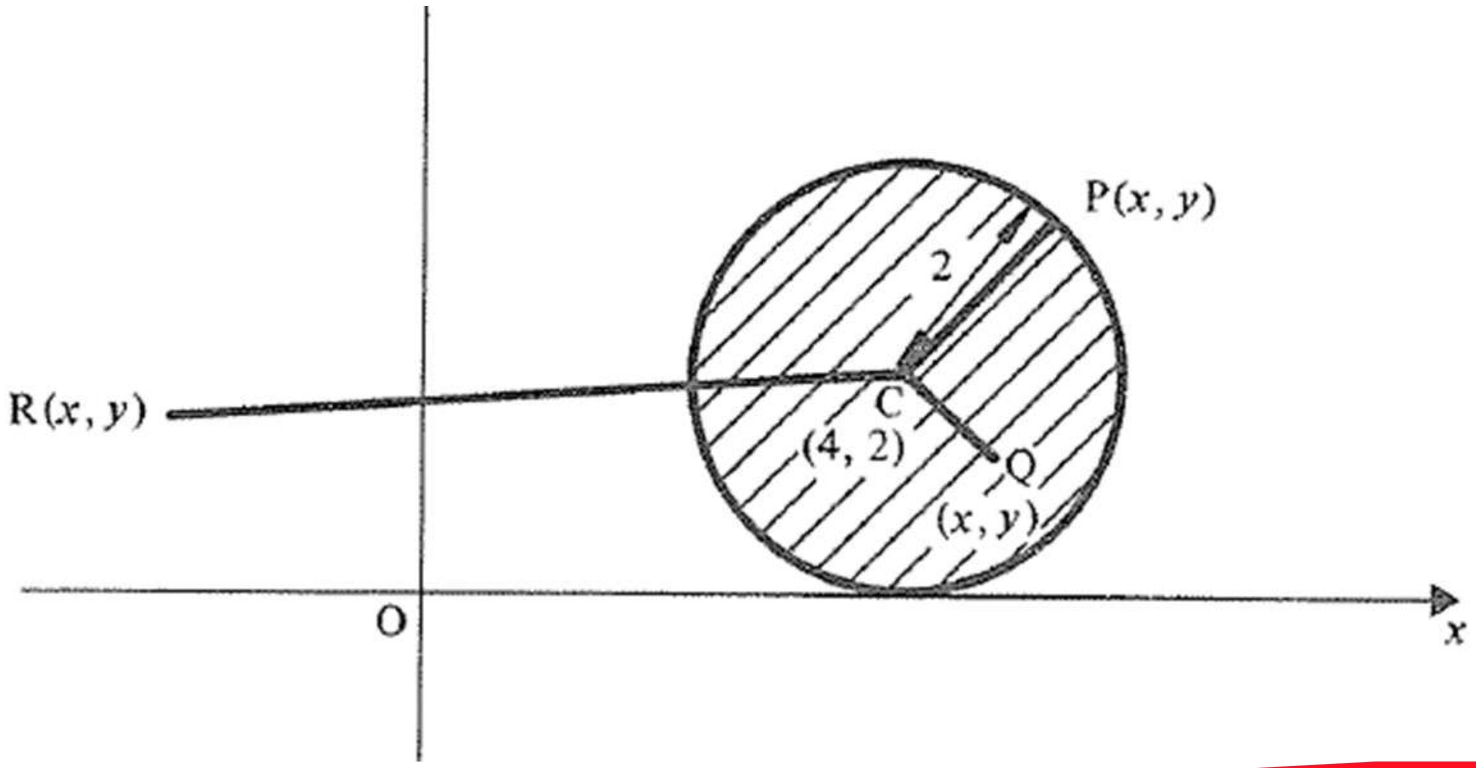


PERPENDICULAR LINES



EQUATION OF A CIRCLE

- Consider for example the circle whose centre is at point (4,2) and whose radius is 2.



Any point P on the circumference of this circle is such that $PC = 2$.
Any point Q inside the circle satisfies the inequality $CQ < 2$.
Any point R outside the circle satisfies the inequality $CR > 2$.
The distance of any point (x, y) from C is given by

$$\sqrt{(x-4)^2 + (y-2)^2}$$

Therefore the coordinates (x, y) of P must satisfy the equation

$$\sqrt{(x-4)^2 + (y-2)^2} = 2$$

or

$$(x-4)^2 + (y-2)^2 = 4$$

Therefore this equation defines the set of points on the circumference of the circle and so is the equation of the circle.

Similarly the coordinates (x, y) of Q satisfy the inequality

$$(x-4)^2 + (y-2)^2 < 4$$

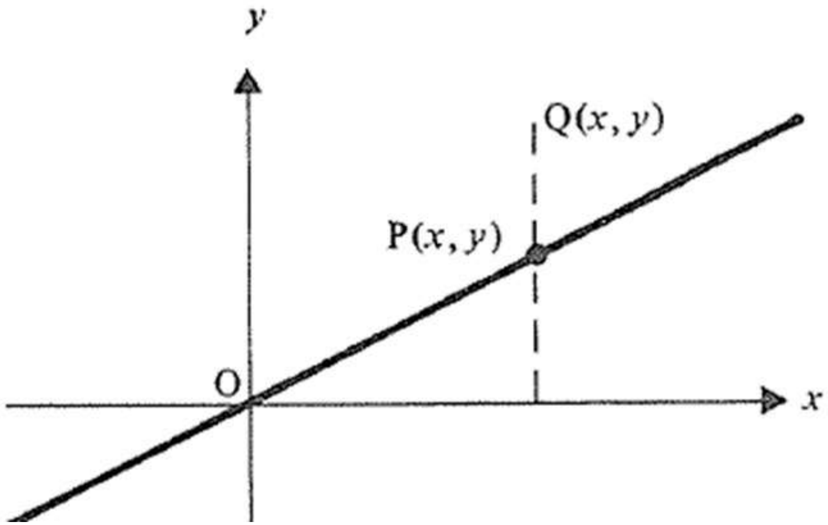
So this inequality defines the region inside the circle and the inequality

$$(x-4)^2 + (y-2)^2 > 4$$

defines the region outside the circle.

EQUATION OF A STRAIGHT LINE

- Straight lines play an important part in any geometric analysis and we will now concentrate our attention to these.
- The equation of a line can be found as follows:



If $P(x, y)$ is any point on the line, the gradient of $OP = \frac{1}{2}$.

The gradient of OP is given by $\dfrac{y-0}{x-0} = \dfrac{y}{x}$

Therefore the coordinates of P satisfy the equation

$$\frac{y}{x} = \frac{1}{2} \quad \text{or} \quad 2y = x$$

Therefore $2y = x$ is the equation of the line.

Note. For any point $Q(x, y)$ above P, $y > \frac{1}{2}x \Rightarrow 2y > x$.
Therefore the inequality $2y > x$ defines the region above the line.
Similarly $2y < x$ defines the region below the line.

EQUATION OF A TANGENT TO A CIRCLE

Given the point $P(2;-4)$ on the circle $(x-4)^2+(y+5)^2=5$. Find the equation of the tangent at P.

Solution:

Given:

- the equation for the circle $(x-4)^2+(y+5)^2=5$
- a point on the circumference of the circle $P(2;-4)$

Required:

- the equation of the tangent in the form $y=mx+c$

The coordinates of the centre of the circle are $(a;b)=(4;-5)$.

The gradient of the radius:

$$m_r = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{-4 - (-5)}{2 - 4}$$

$$= -\frac{1}{2}$$

$$m \times m_{\perp} = -1$$

$$\therefore m_{\perp} = -\frac{1}{m_r}$$

$$= \frac{1}{\frac{1}{2}}$$

$$= 2$$

Equation of the tangent:

$$
\begin{aligned}
y &= m_{\perp}x + c \\
-4 &= 2(2) + c \\
c &= -8
\end{aligned}
$$

The equation of the tangent to the circle is

$$y = 2x - 8$$

EQUATION OF A CIRCLE

a) Find the equation of the circle with centre $(-3;-2)$ which passes through $(1;-4)$

Solution:

$$(x+3)^2+(y+2)^2=r^2$$
$$(1+3)^2+(-4+2)^2=r^2$$
$$(4)^2+(-2)^2=r^2$$
$$16+4=r^2$$
$$20=r^2$$
$$(x+3)^2+(y+2)^2=20$$

GEOMETRY THEOREMS FOR GR 11 AND 12

- The angle at the centre is twice the angle at the circumference subtended by the same arc.
- The tangent to a circle is perpendicular to the radius drawn to the point of contact and conversely.
- The perpendicular from the centre of a circle to a chord bisects the chord.
- The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.
- The perpendicular bisector of a chord passes through the centre of the circle

- Equal chords in equal circles are equidistant from the centres.
- Chords in a circle which are equidistant from the centre are equal
- Any three non-collinear points lie on a unique circle, whose centre is the point of concurrency of the perpendicular bisectors of the intervals joining the points.
- Angles in the same segment are equal.
- The angle in a semicircle is a right angle.

- Opposite angles of a cyclic quadrilateral are supplementary
- The exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle.
- If the opposite angles in a quadrilateral are supplementary then the quadrilateral is cyclic.
- The products of the intercepts of two intersecting chords are equal.
- The products of the intercepts of two intersecting secants to a circle from an external point

- Tangents to a circle from an external point are equal
- The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.
- The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point.
- When two circles intersect, the line joining their centres bisects their common chord at right angles
- Equal arcs on circles of equal radii subtend equal angles at the centre, and conversely
- Equal angles at the centre stand on equal chords, and conversely

GRADE 11 QUESTIONS

EXERCISE

In the diagram, O is the centre of the circle. Diameter LR subtends $L\hat{K}R$ at the circumference of the circle. N is another point on the circumference and chords LN and KN are drawn. $\hat{L}_1 = 58°$.

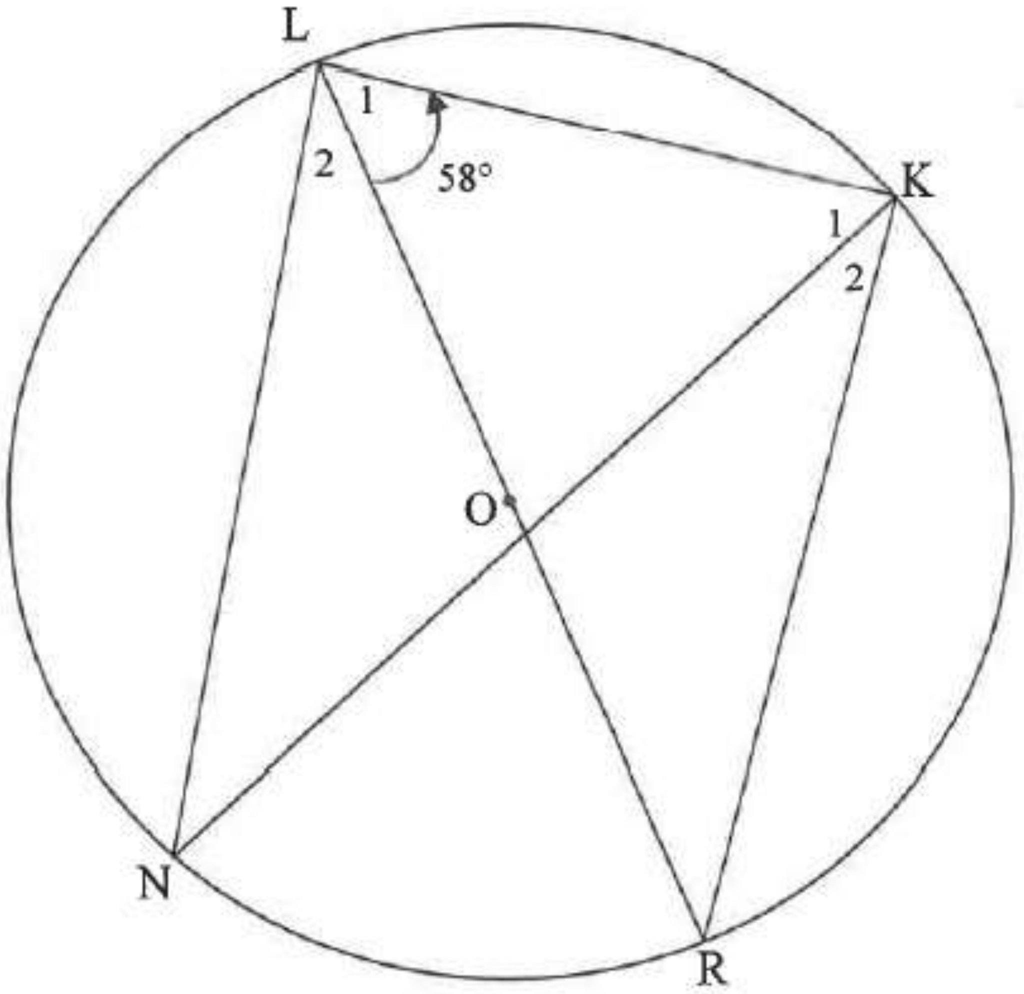
Calculate, giving reasons, the size of:

9.1 $L\hat{K}R$ (2)

9.2 $\hat{R}$ (2)

9.3 $\hat{N}$ (2)

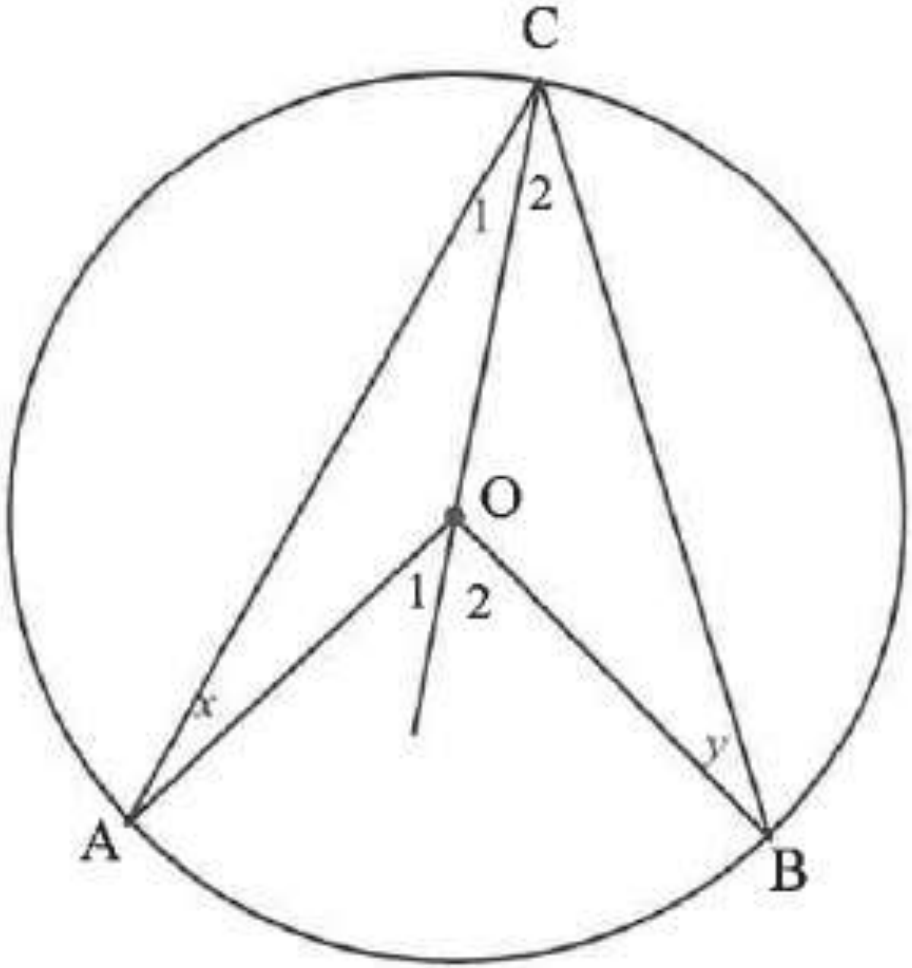
[6]


L
1
2
58°
K
1
2
O
N
R

In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chords AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.

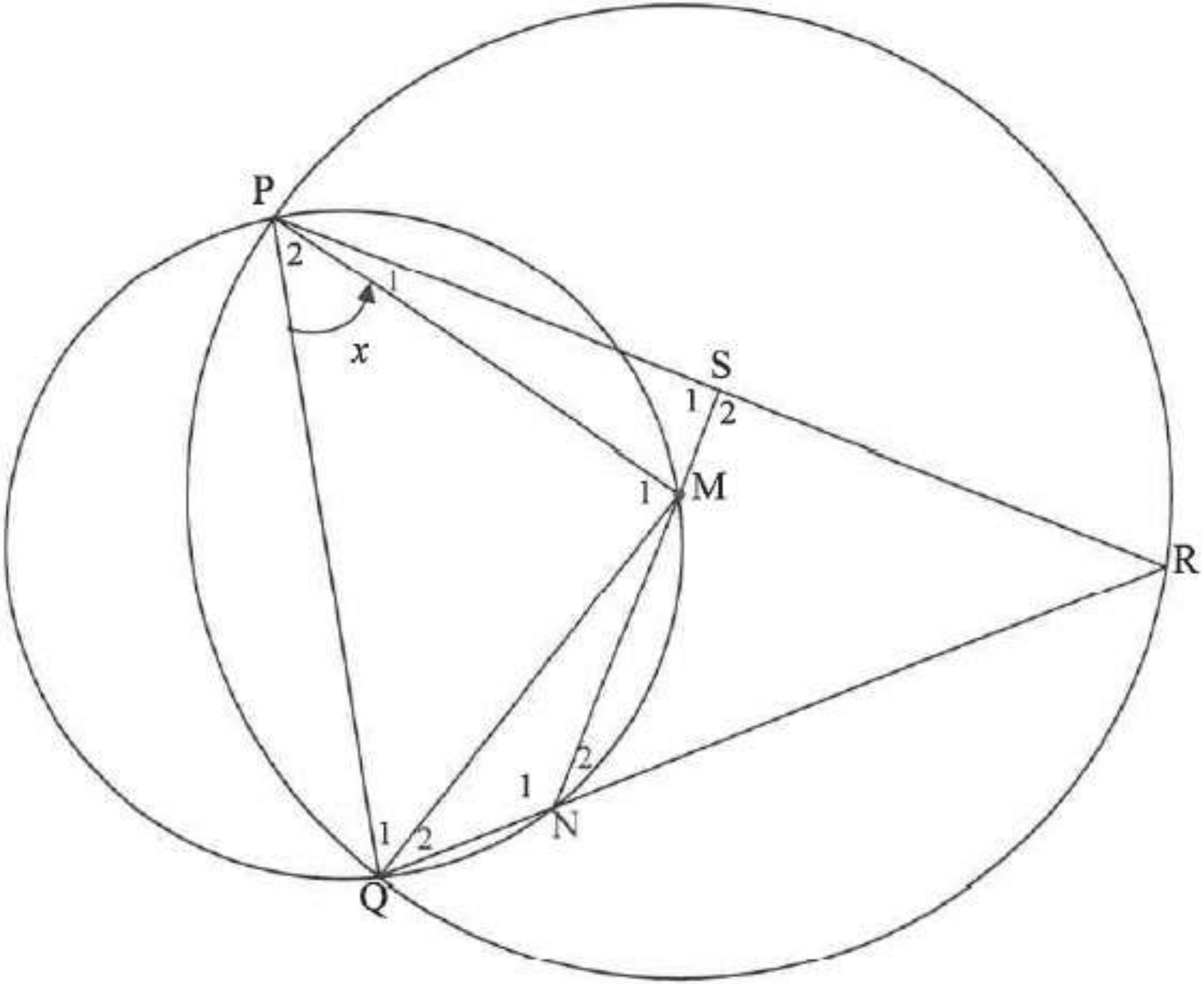
10.1.1 Determine the size of $\hat{O}_1$ in terms of x. (3)

10.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $A\hat{O}B = 2A\hat{C}B$. (3)


C
1
2
O
1
2
α
y
A
B

In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.

10.2.1	Give a reason why $\hat{N}_2 = x$.	(1)
10.2.2	Write down another angle equal in size to x. Give a reason.	(2)
10.2.3	Determine the size of $\hat{R}$ in terms of x.	(3)
10.2.4	Prove that PS = SR.	(3)
		[15]


P
2
1
x
S
1
2
1
M
R
2
1
1
2
N
Q

9.1 Complete the statement so that it is TRUE:

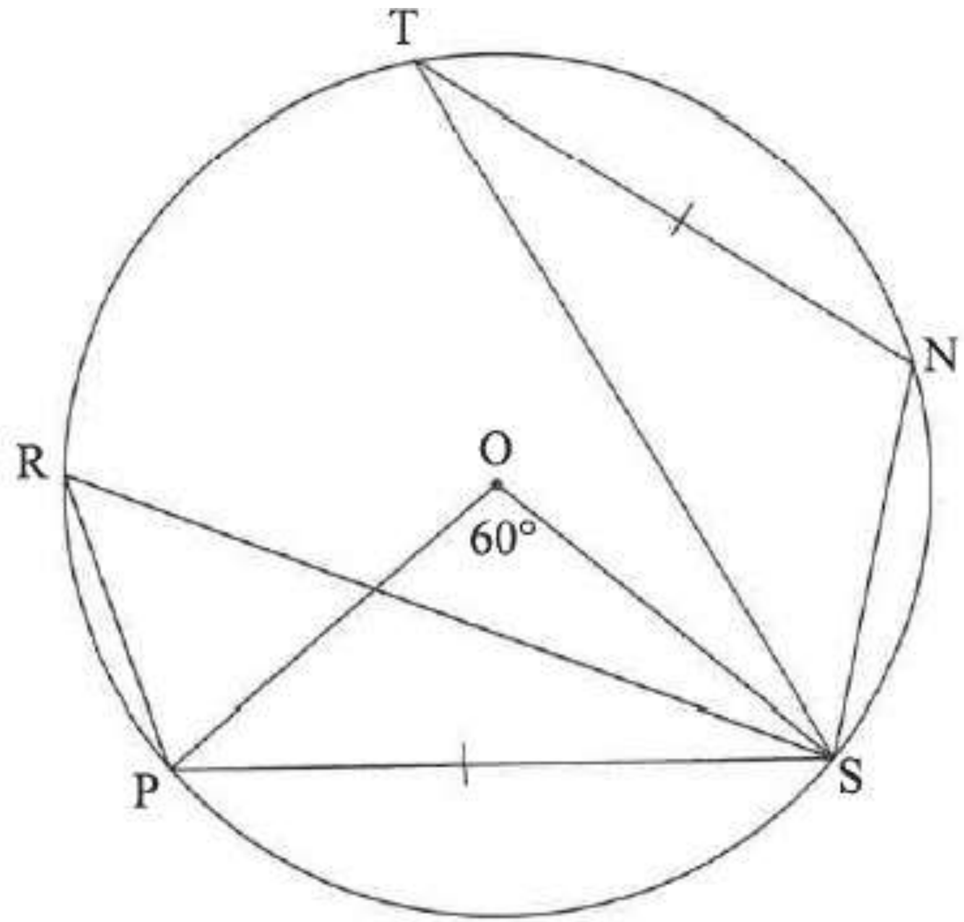
The angle subtended by an arc at the centre of a circle is ...

9.2 O is the centre of circle TNSPR. $P\hat{O}S = 60°$ and $PS = NT$.

Calculate the size of:

9.2.1 $P\hat{R}S$

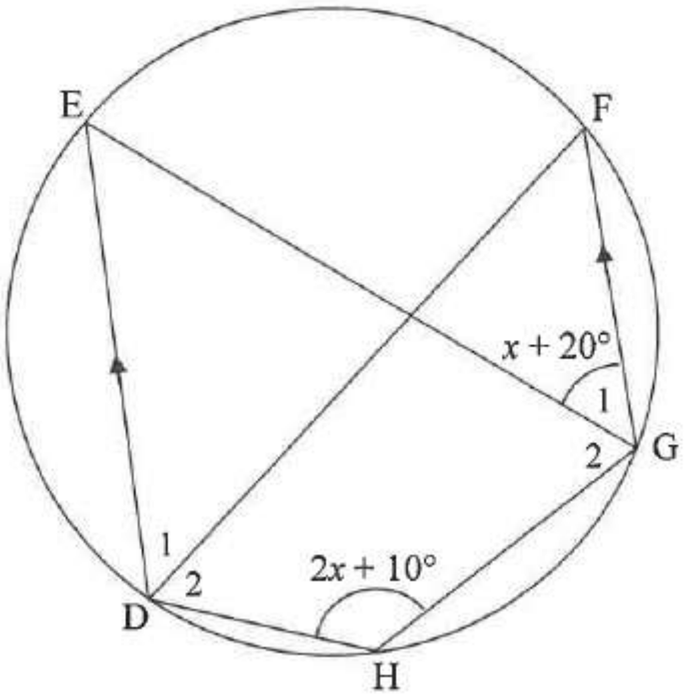
9.2.2 $N\hat{S}T$


T
N
R
O
60°
P
S

D, E, F, G and H are points on the circumference of the circle.
$\hat{G}_1 = x + 20°$ and $\hat{H} = 2x + 10°$. DE || FG.

10.1 Determine the size of $D\hat{E}G$ in terms of x.

10.2 Calculate the size of $D\hat{H}G$.

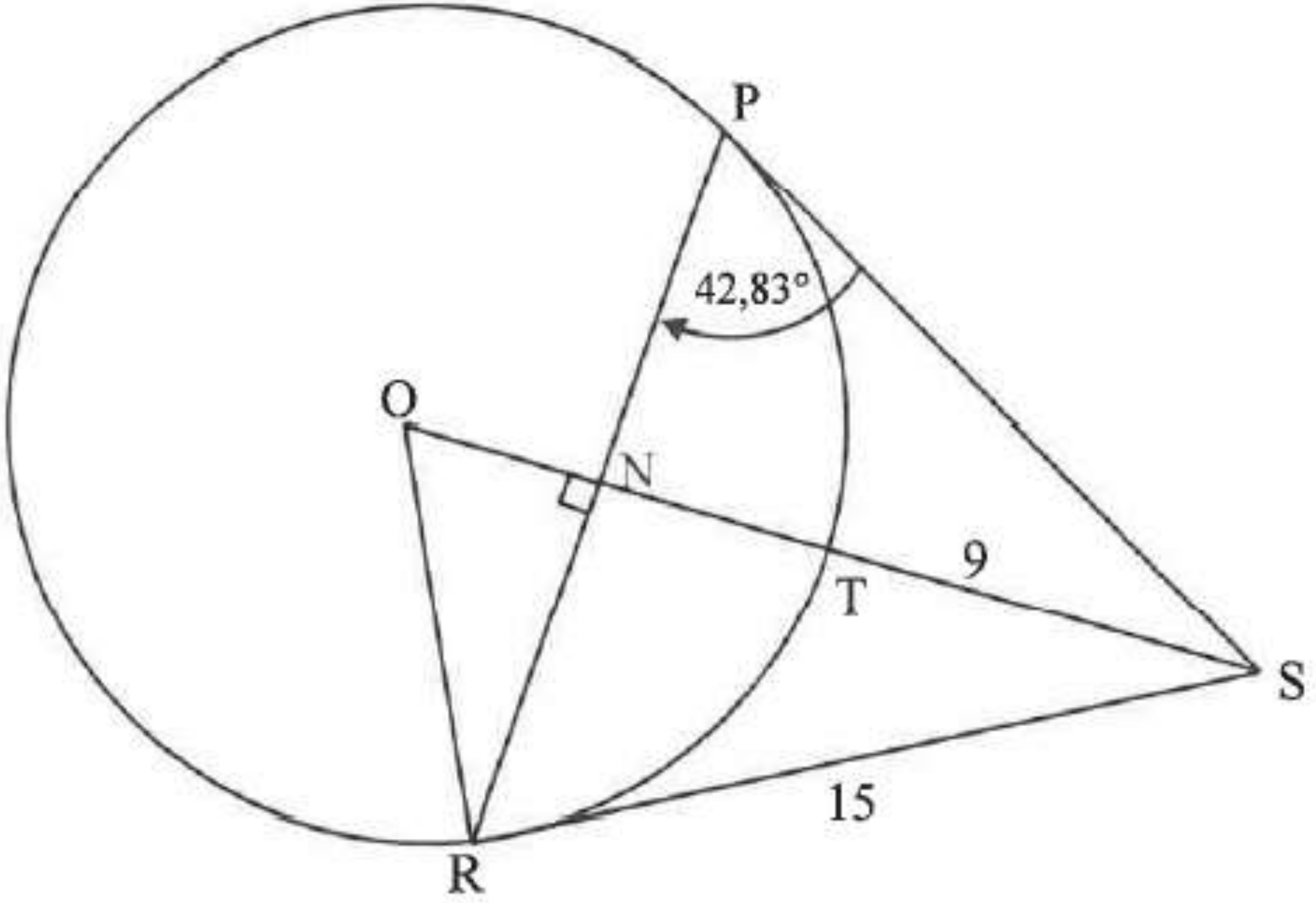

E
F
x + 20°
1
2
G
1
2
2x + 10°
D
H

O is the centre of the circle PTR. N is a point on chord RP such that $ON \perp PR$.
RS and PS are tangents to the circle at R and P respectively.

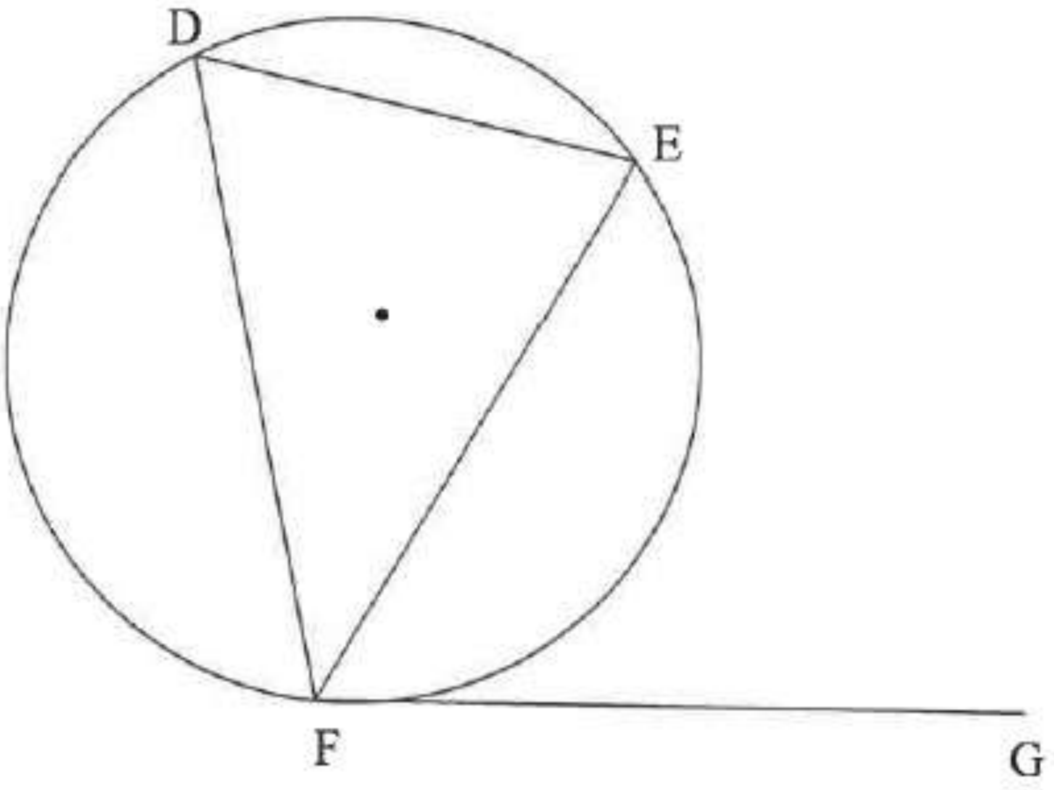
$RS = 15$ units; $TS = 9$ units; $R\hat{P}S = 42{,}83°$.

11.1 Calculate the size of $N\hat{O}R$.

11.2 Calculate the length of the radius of the circle.


P
42,83°
O
N
9
T
S
15
R

12.1 Use the diagram below to prove the theorem which states that $E\hat{F}G = E\hat{D}F$.



GRADE 12 QUESTIONS

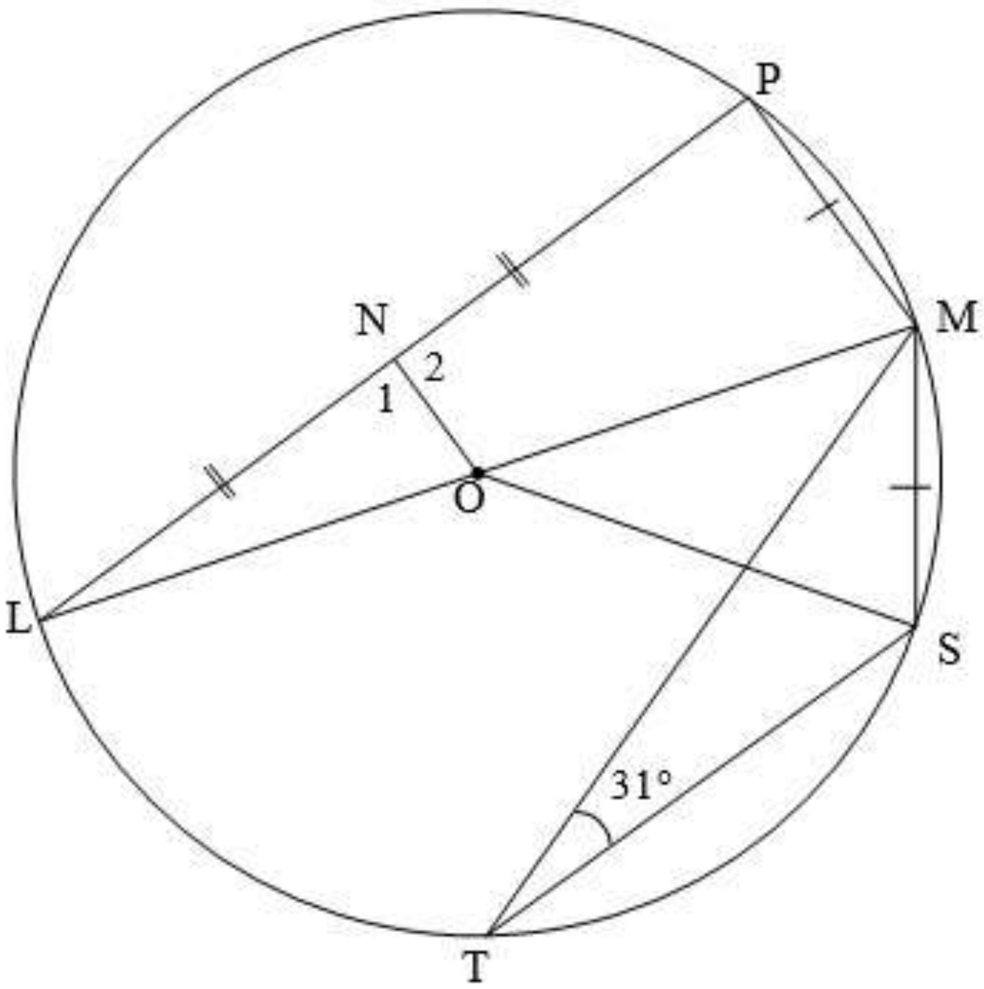
8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $M\hat{T}S = 31°$

8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $M\hat{O}S$ (2)

(b) $\hat{L}$ (2)

8.1.2 Prove that $ON = \frac{1}{2}MS$. (4)


P
N
M
2
1
O
L
S
31°
T

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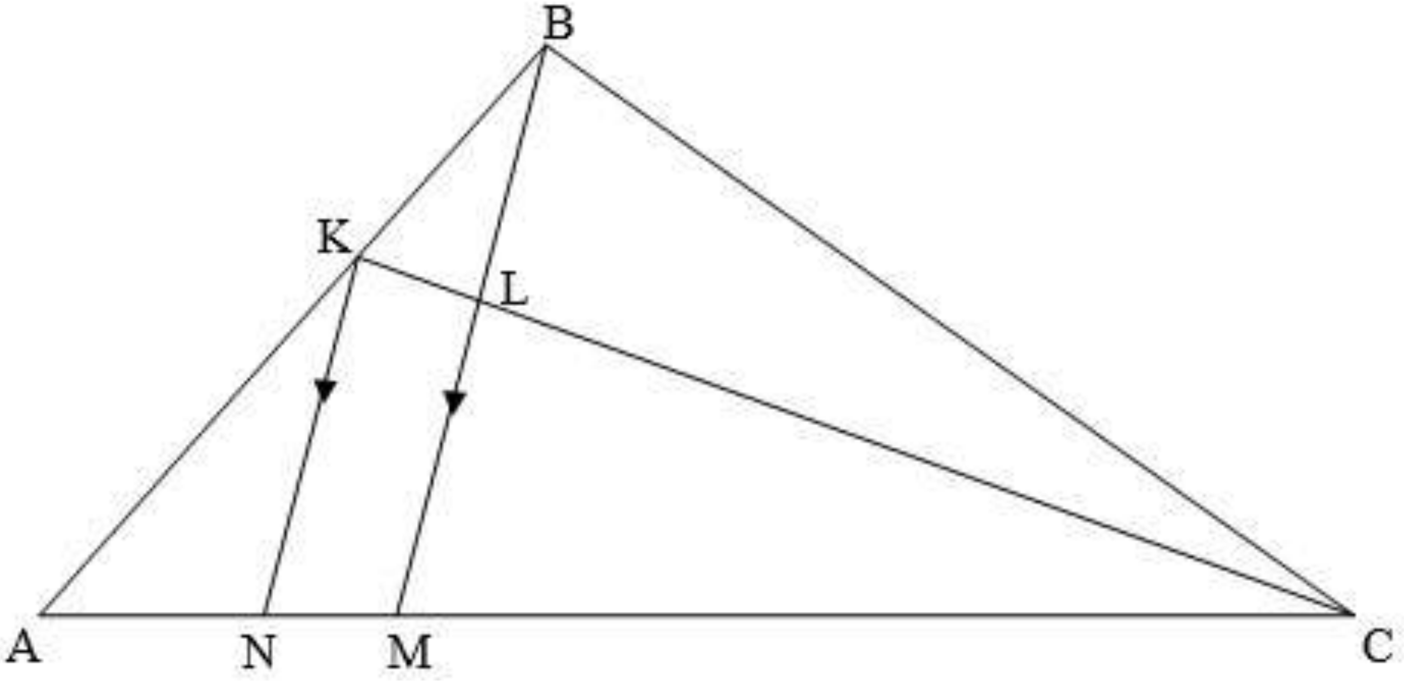
8.2 In ΔABC in the diagram, K is a point on AB such that $AK : KB = 3 : 2$. N and M are points on AC such that $KN \parallel BM$. BM intersects KC at L. $AM : MC = 10 : 23$.



Determine, with reasons, the ratio of:

8.2.1 $\frac{AN}{AM}$ (2)

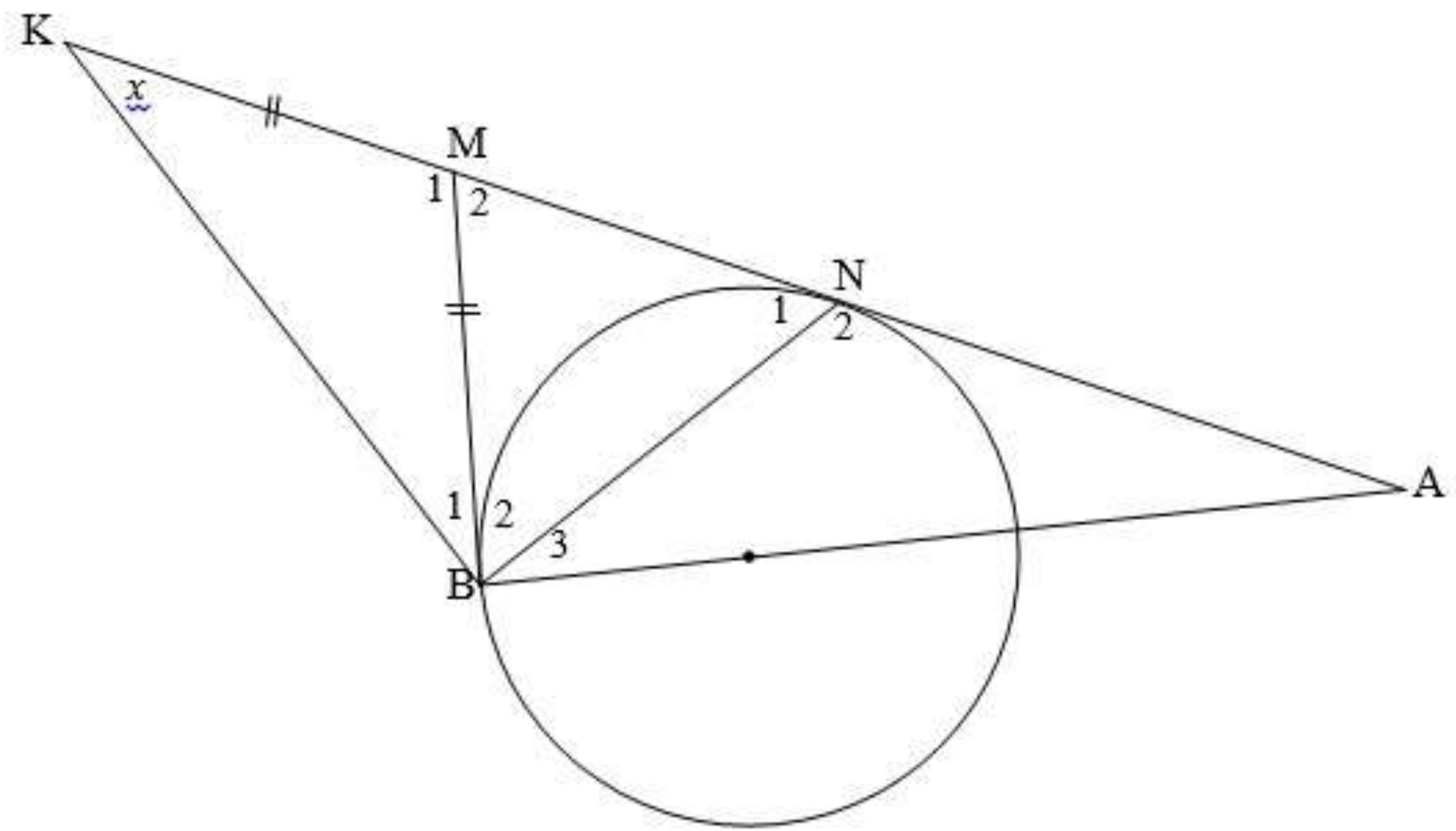
8.2.2 $\frac{CL}{LK}$ (3)

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N. The straight line from B passing through the centre of the circle meets MN produced in A. NM is produced to K such that BM = MK. BK and BN are drawn.

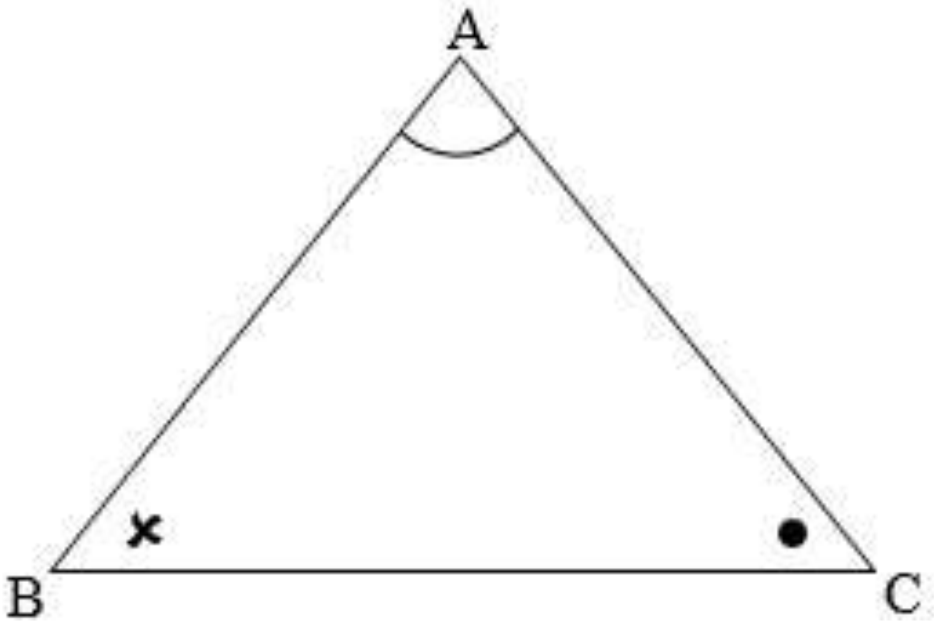
Let $\hat{K} = x$.

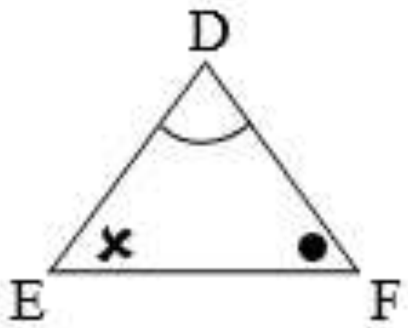
9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x. (6)

9.2 Prove that BA is a tangent to the circle passing through K, B and N. (5)


K
x
M
1
2
N
1
2
1
2
3
B
A

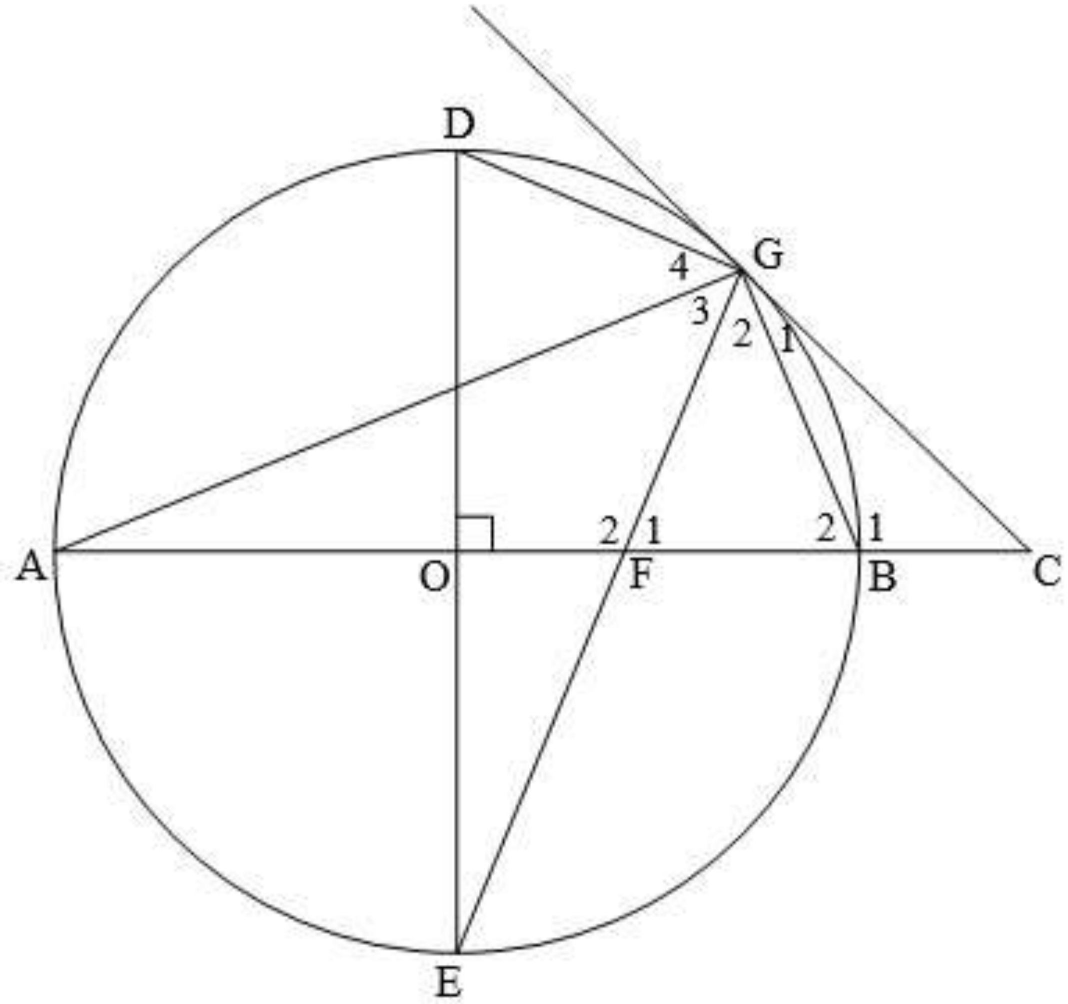
10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$





Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G. The straight line from C passing through O cuts the circle at A and B. Diameter DOE is perpendicular to CA. GE and CA intersect at F. Chords DG, BG and AG are drawn.


D
G
4
3
2
1
A
O
2
1
F
2
1
B
C
E

10.2.1 Prove that:

(a) DGFO is a cyclic quadrilateral (3)

(b) GC = CF (5)

10.2.2 If it is further given that CO = 11 units and DE = 14 units, calculate:

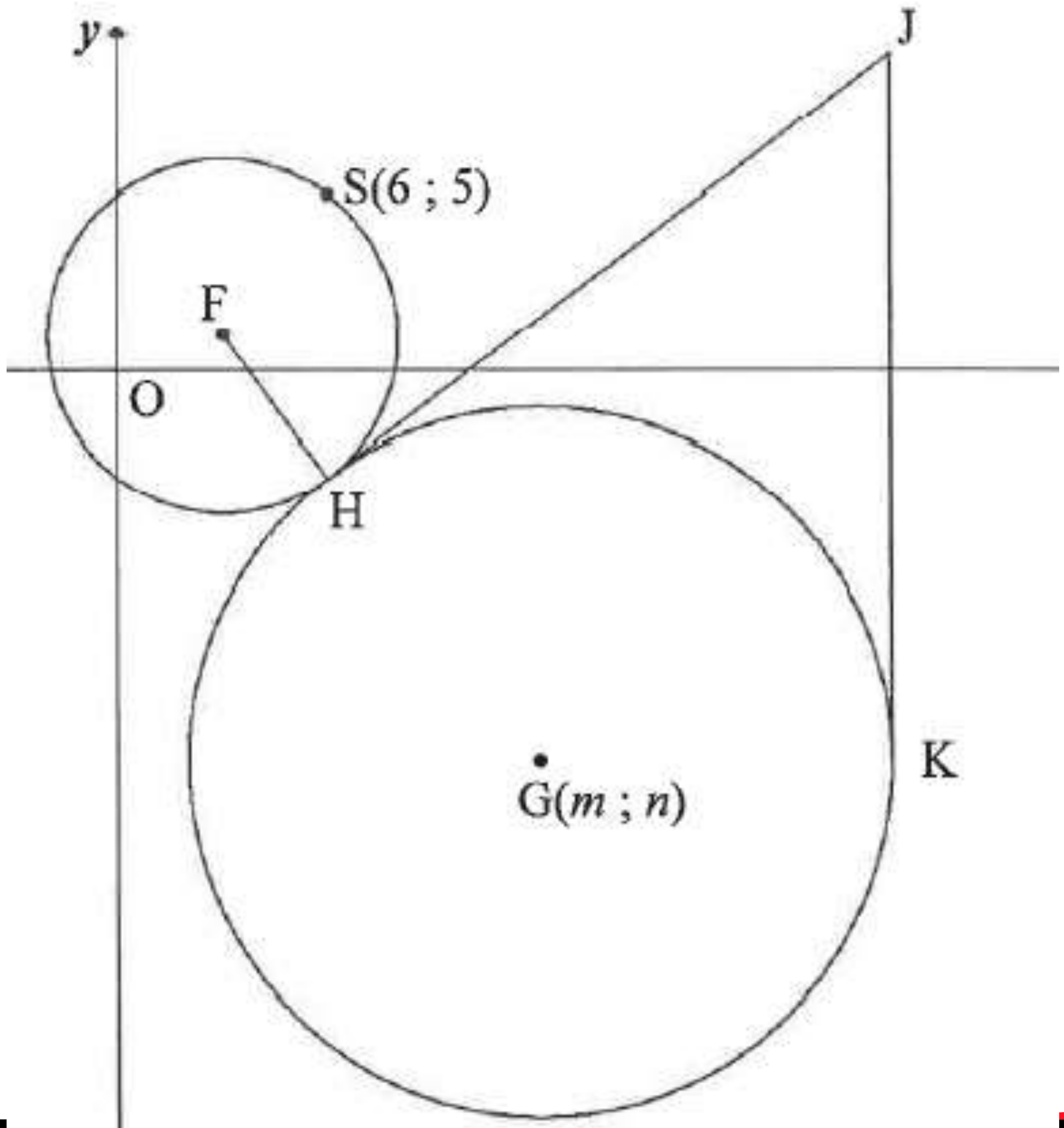
(a) The length of BC (3)

(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G(m ; n) in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.

4.1	Write down the coordinates of F.	(2)
4.2	Calculate the length of FS.	(2)
4.3	Write down the length of HG.	(1)
4.4	Give a reason why JH = JK.	(1)


y
J
S(6 ; 5)
F
O
H
K
G(m ; n)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that $JK = 20$ (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x = 22$ (7)

In the diagram, a circle having centre M touches the x–axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.

4.1 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

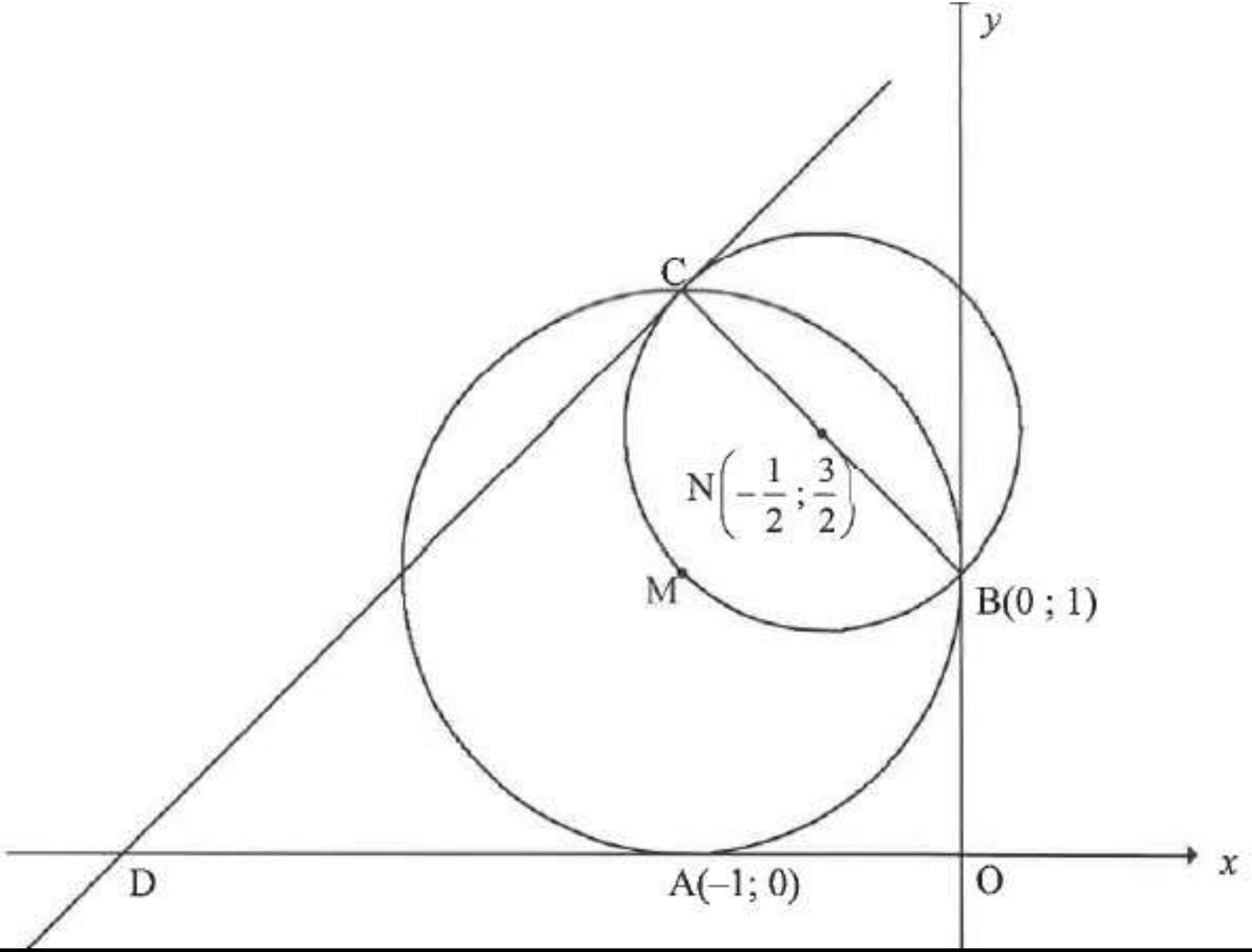
4.2 Calculate the coordinates of C. (2)

4.3 Show that the equation of the tangent CD is $y-x=3$. (4)

4.4 Determine the values of t for which the line $y = x + t$ will NOT touch or cut the smaller circle. (3)

4.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle. (3)

4.6 If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a > 0$, show that $a = \frac{\sqrt{7}}{2}$ units. (5)


y
C
N(−1/2 ; 3/2)
M
B(0 ; 1)
D
A(−1; 0)
O
x

8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.

Calculate, with reasons, the size of EACH of the following angles:

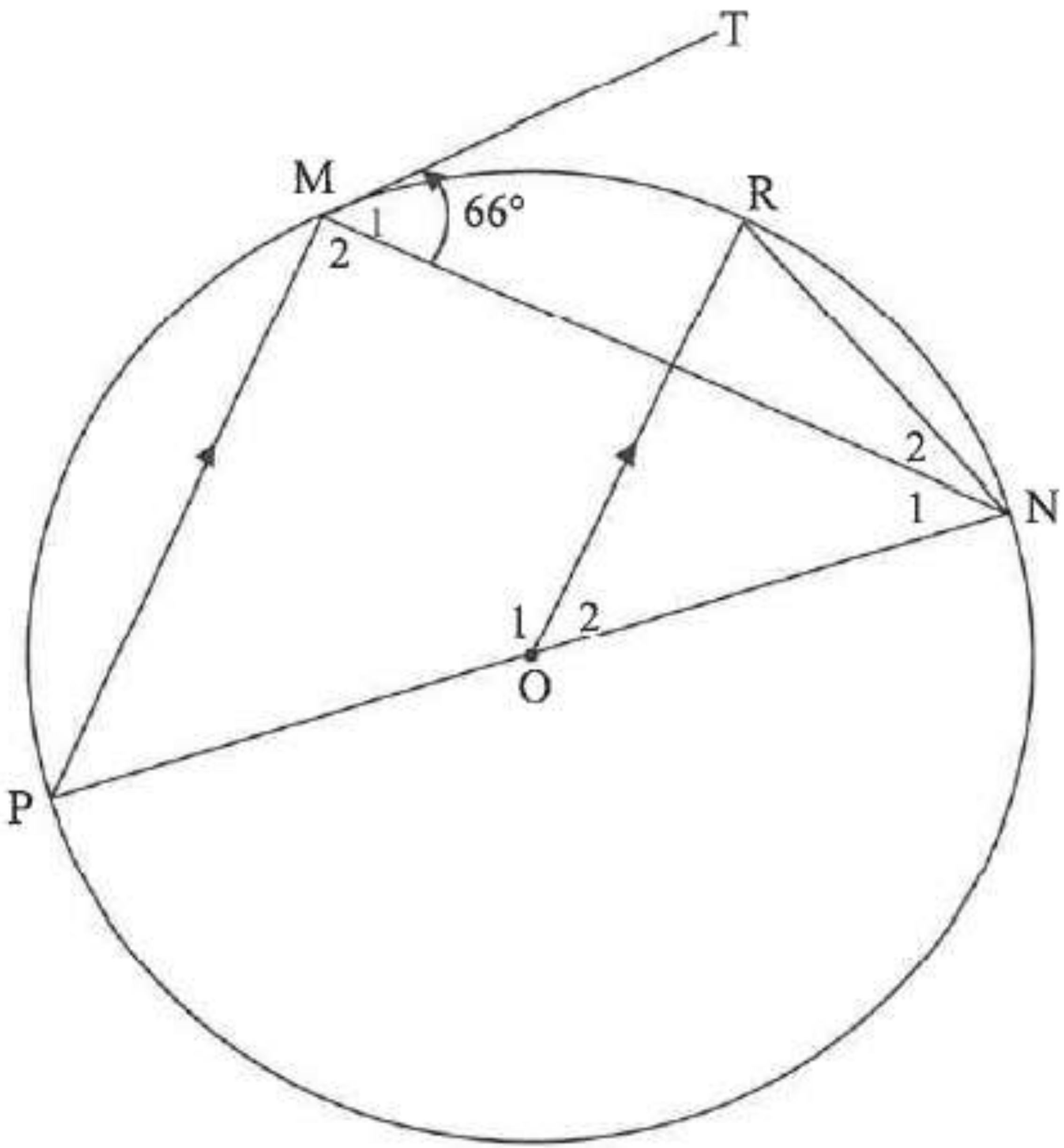
8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_2$ (2)

8.1.3 $\hat{N}_1$ (1)

8.1.4 $\hat{O}_2$ (2)

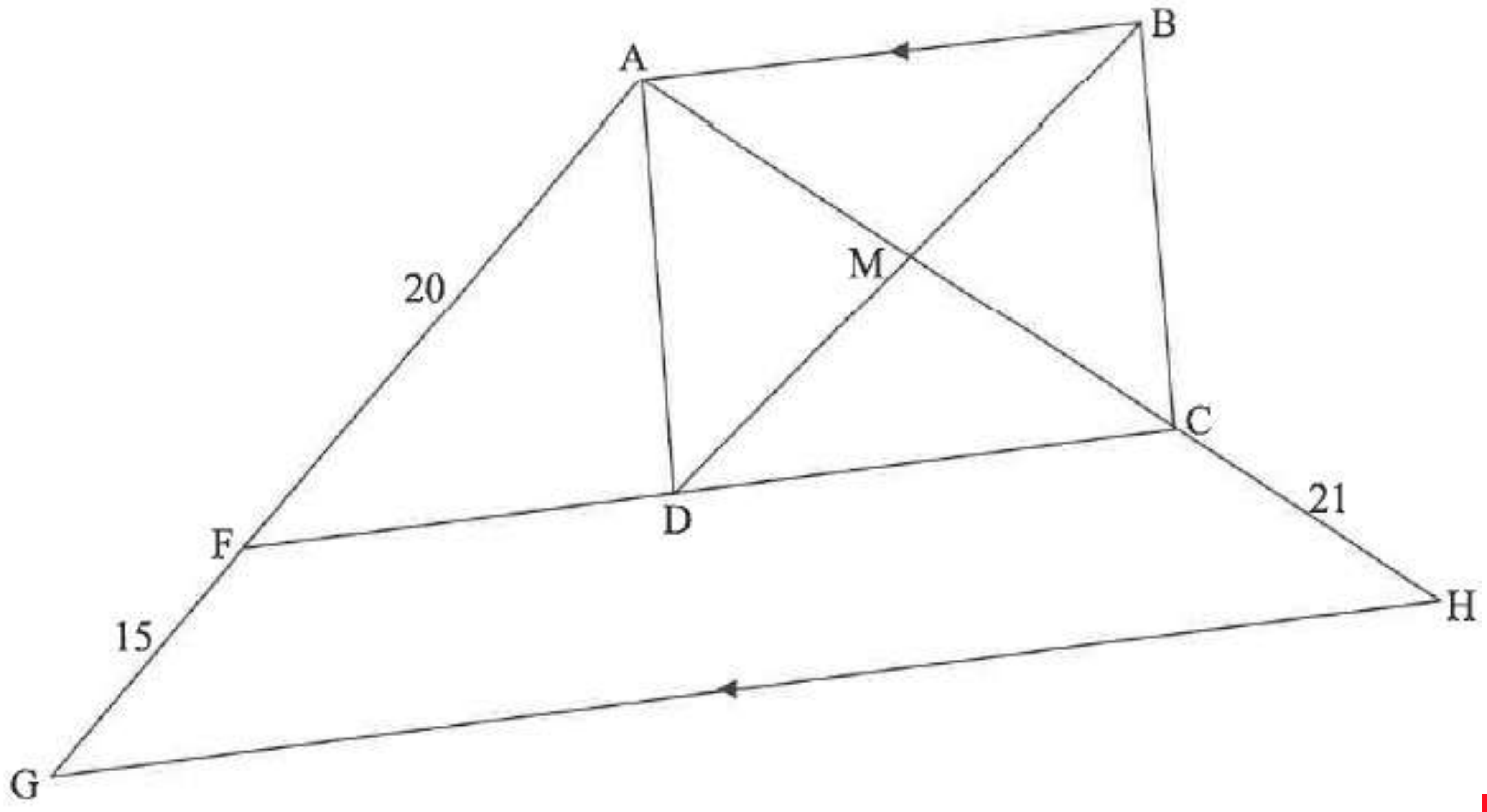
8.1.5 $\hat{N}_2$ (3)


T
M
R
66°
1
2
2
1
N
1
2
O
P

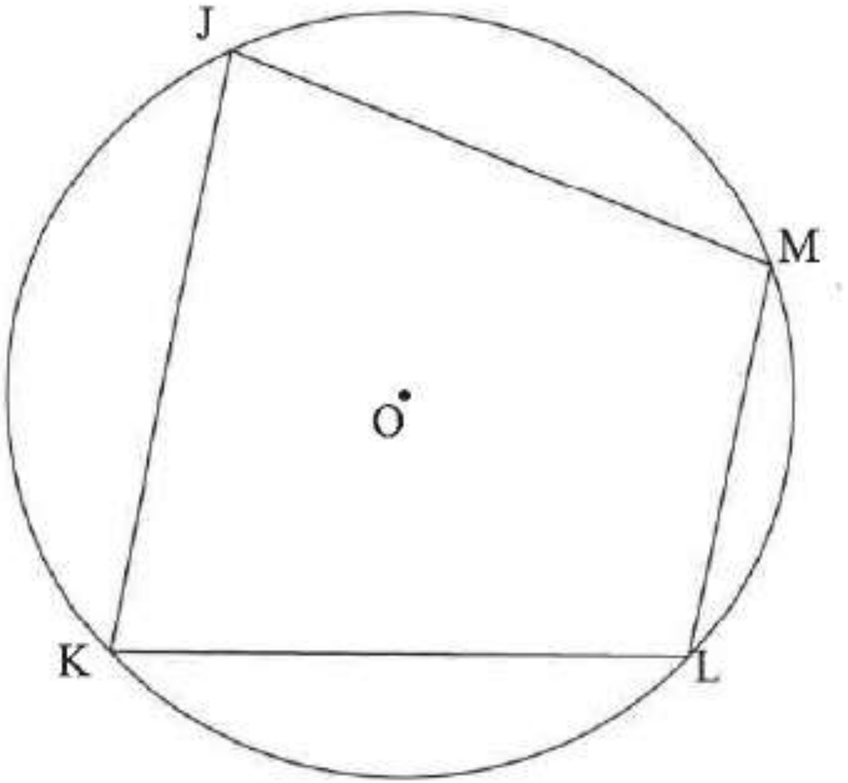
8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.

8.2.1 Explain why FC $\parallel$ GH. (1)

8.2.2 Calculate, with reasons, the length of DM. (5)


A
B
M
20
C
21
D
F
H
15
G

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.

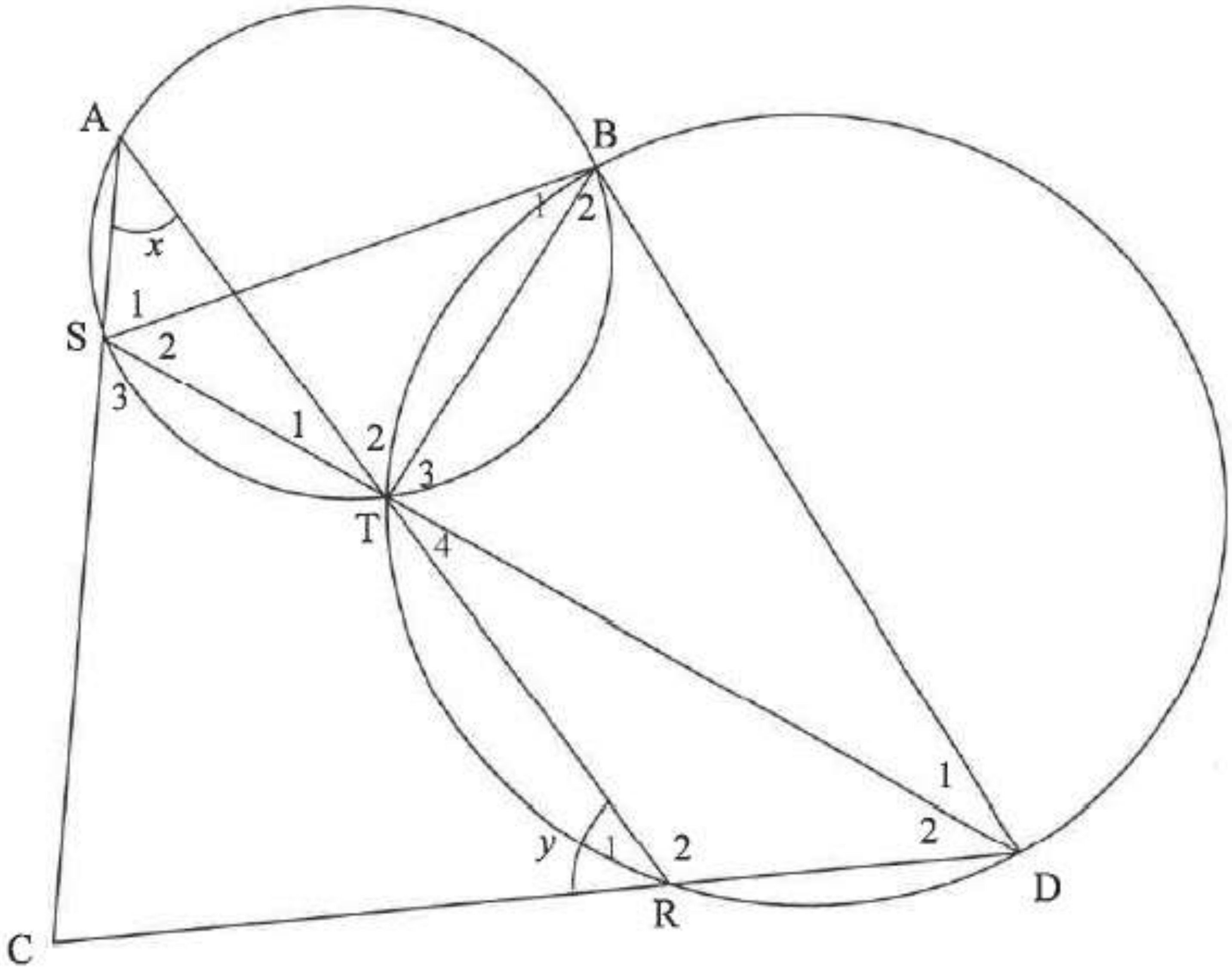
9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)


A
B
x
1
2
S
1
2
3
1
2
3
T
4
1
2
y
1
2
R
D
C

In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.

10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 $\Delta ACB \parallel\!| \Delta CMD$ (3)

10.2 Hence, or otherwise, prove that:

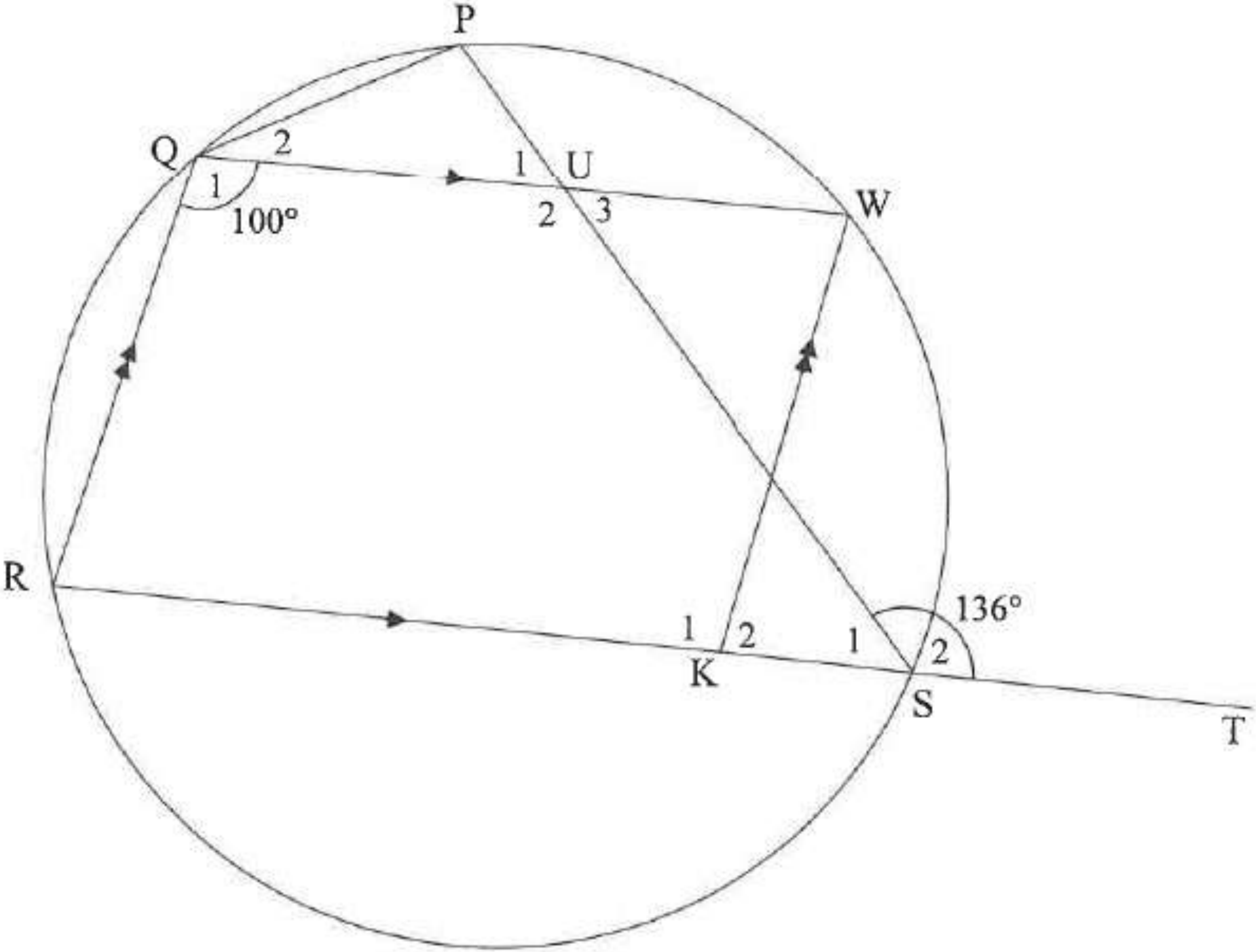
10.2.1 $\dfrac{CM^2}{DC^2} = \dfrac{AM}{AB}$ (6)

10.2.2 $\dfrac{AM}{AB} = \sin^2 x$ (2)

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.

Determine, with reasons, the size of:

8.1.1	$\hat{R}$	(2)
8.1.2	$\hat{P}$	(2)
8.1.3	$P\hat{Q}W$	(3)
8.1.4	$\hat{U}_2$	(2)

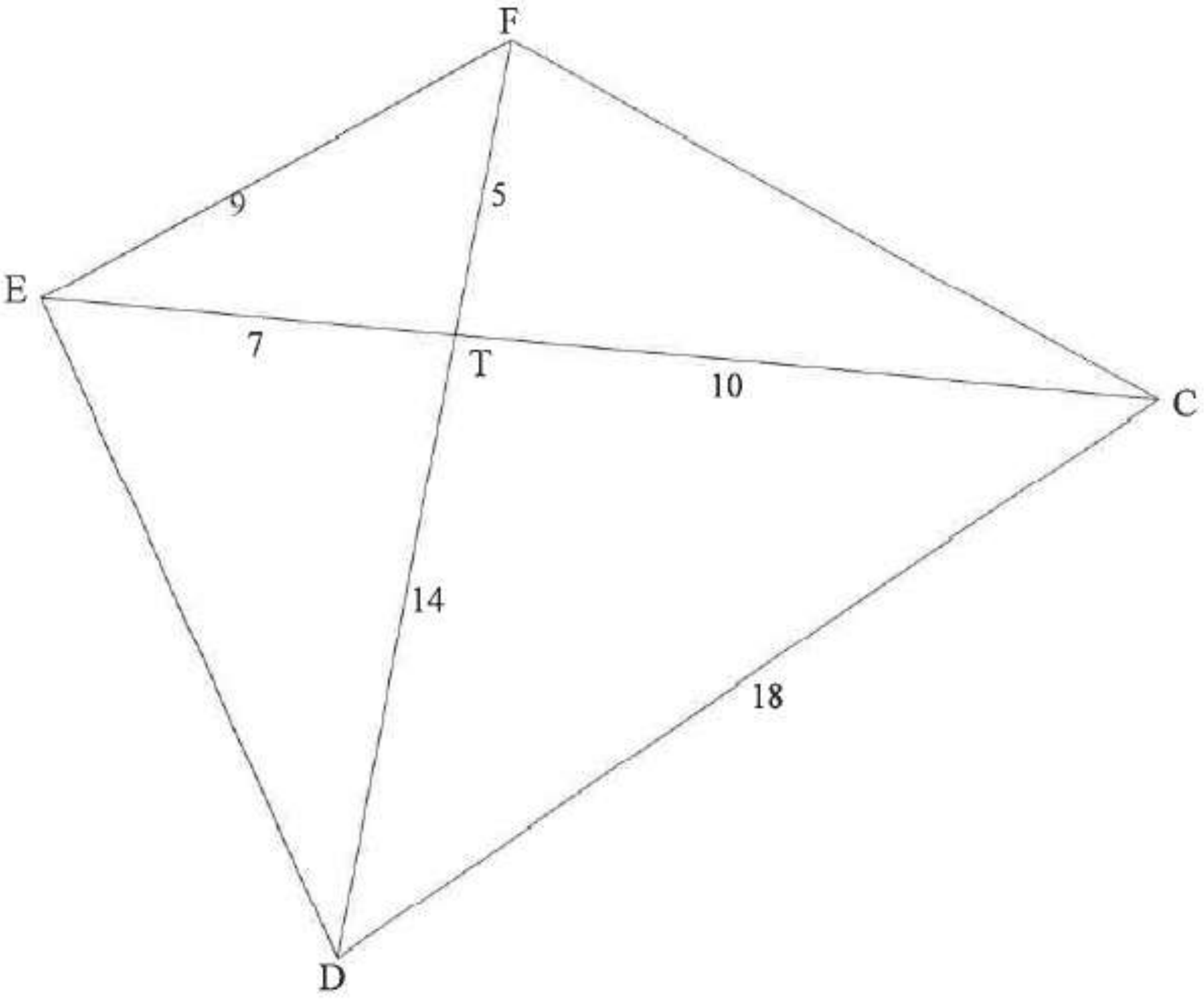

P
Q
2
1
100°
1
U
2
3
W
R
1
2
K
1
136°
2
S
T

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T. EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.

Prove, with reasons, that:

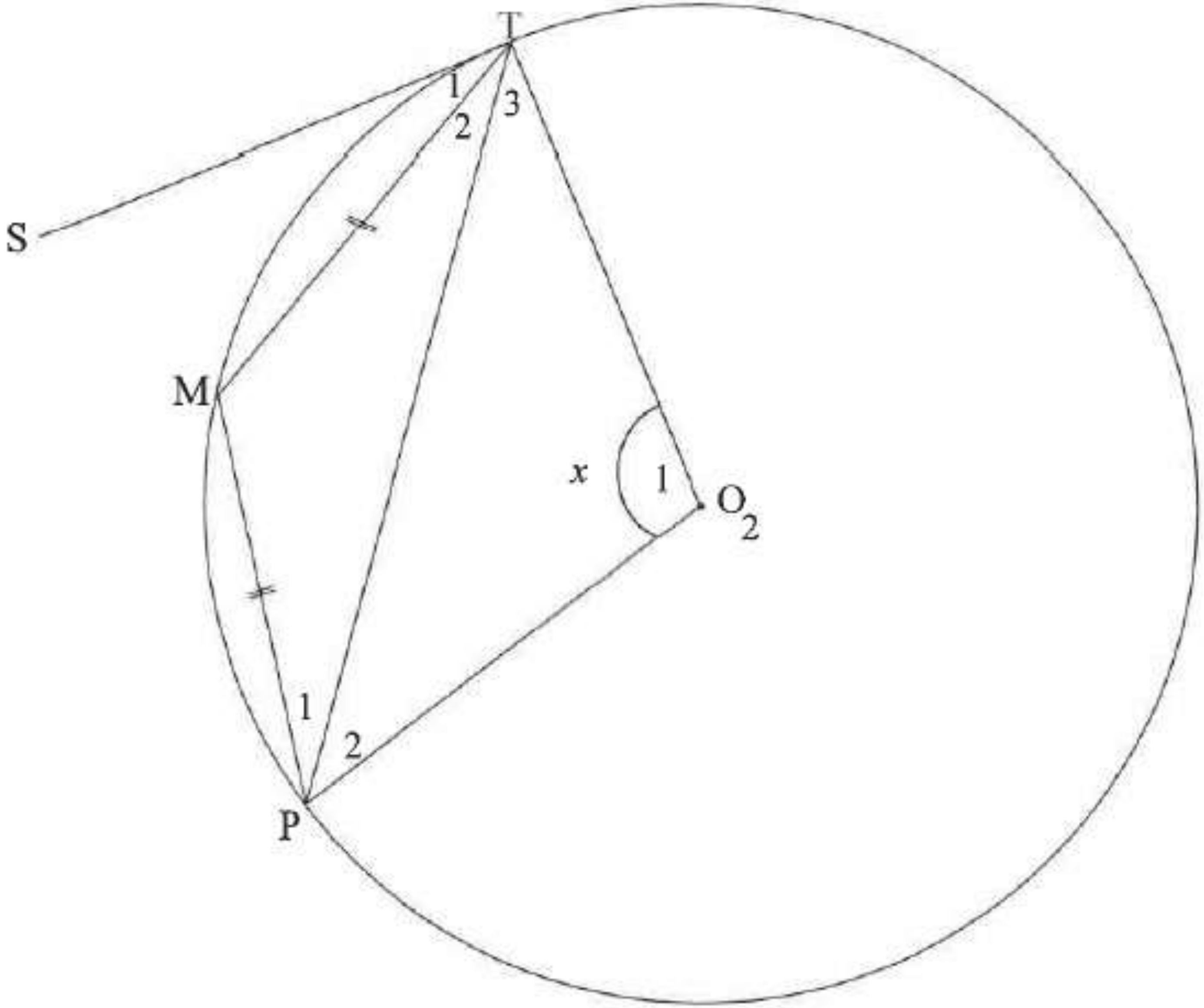
8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)


F
E
T
C
D
9
5
7
10
14
18

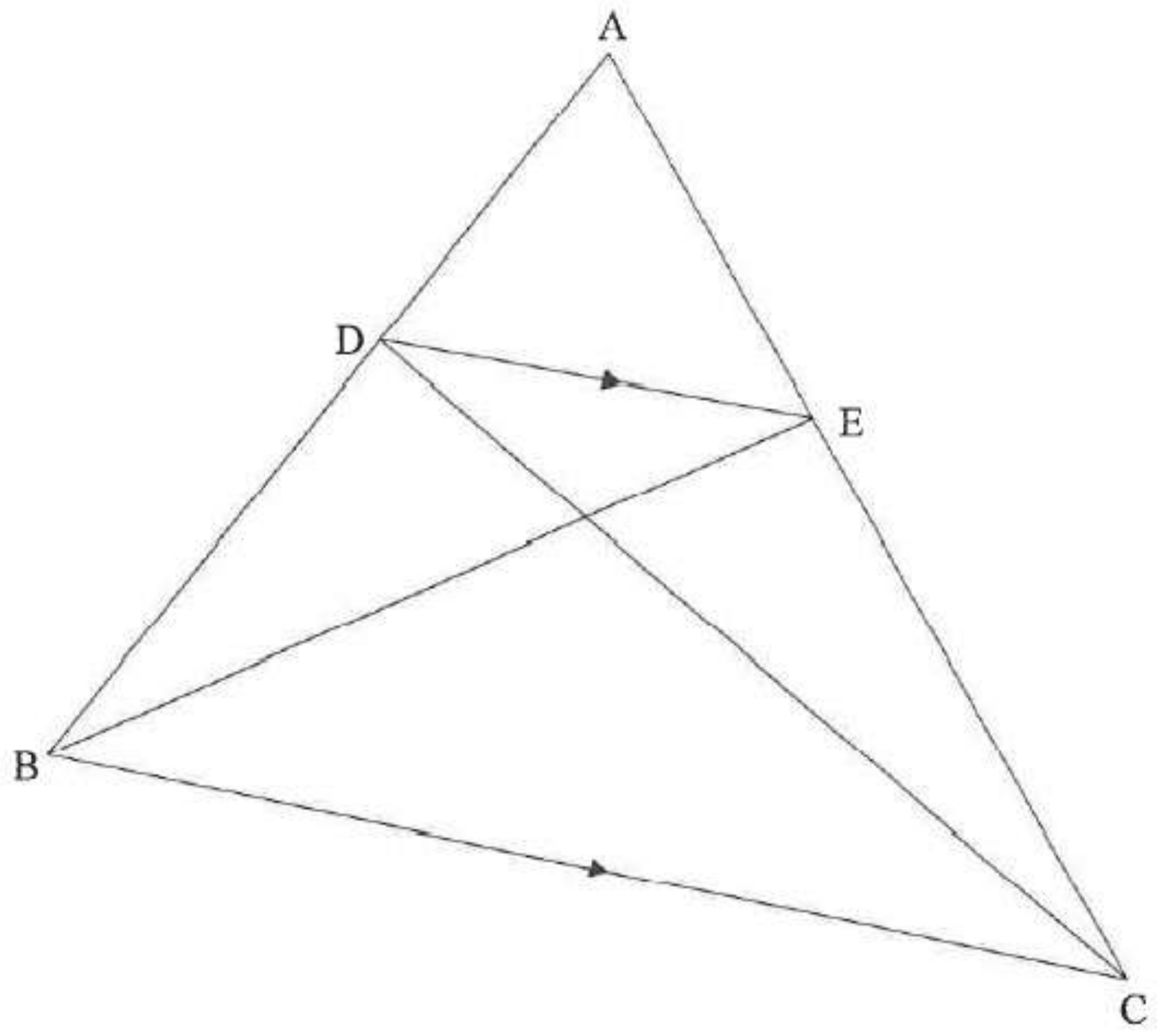
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.

Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$.


T
1
2
3
S
M
x
1
O
2
1
2
P

10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that $DE \parallel BC$. BE and DC are drawn.

Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)


A
D
E
B
C

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.

10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

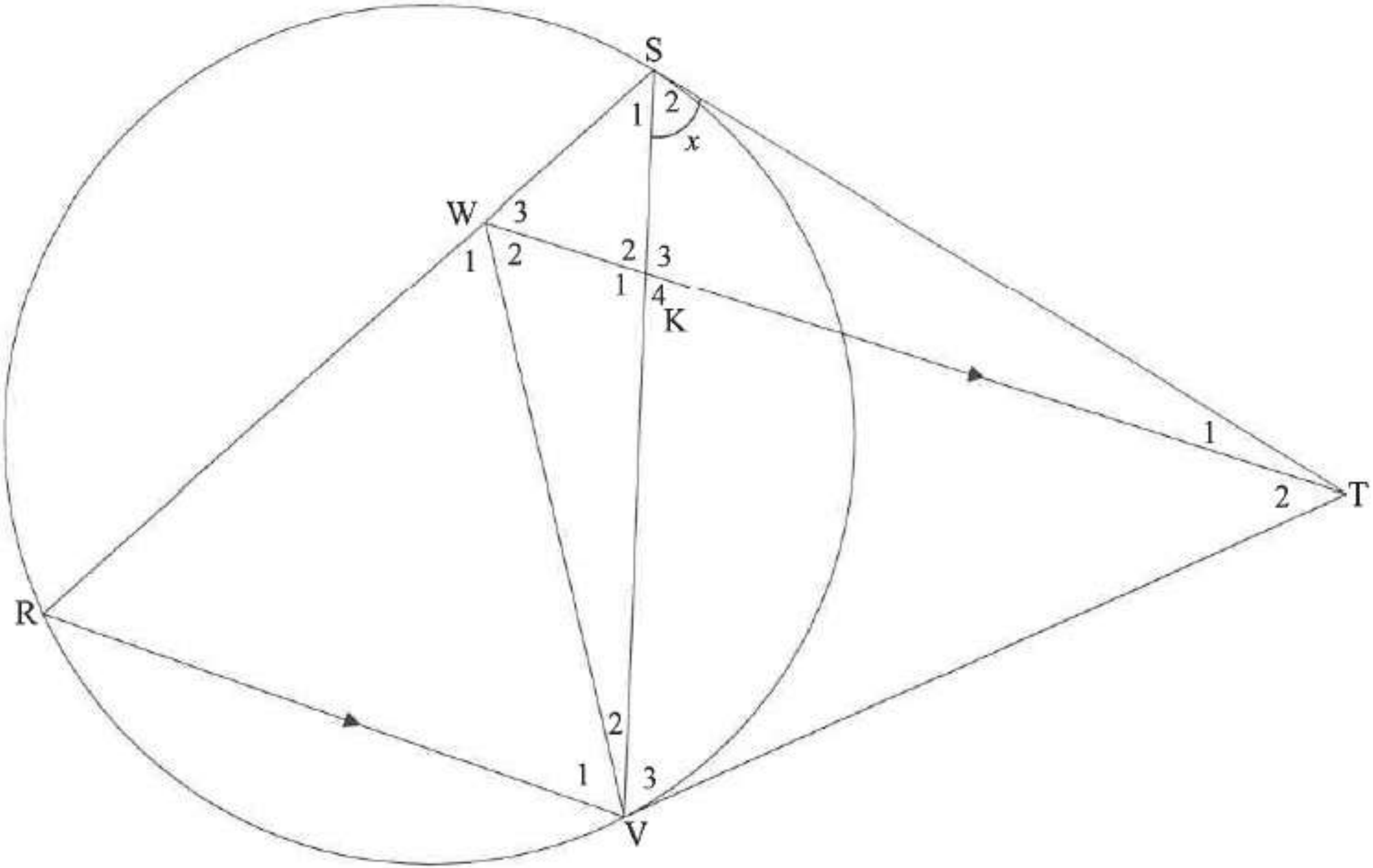
10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

(d) $\dfrac{RV}{SR} = \dfrac{KV}{TS}$ (4)


S
1
2
x
W
3
1
2
2
3
1
4
K
1
2
T
R
2
1
3
V

THANK YOU

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