BELGIUM CAMPUS iTversity

It's the way we're wired

GRADE 12 MATHS

Charmaine

FUNCTIONS

Functions of the general form $y = \frac{a}{x} + q$ where $a, x\ and\ y\ \neq 0$ are called hyperbolic functions.

The effect of q

The effect of q is called a vertical shift because all points are moved the same distance in the same direction (it slides the entire graph up or down).

- For $q > 0$, the graph of $f(x)$ is shifted vertically upwards by q units.
- For $q < 0$, the graph of $f(x)$ is shifted vertically downwards by q units.

The horizontal asymptote is the line $y = q$ and the vertical asymptote is always the y-axis, the line $x = 0$.

FUNCTIONS

The effect of *a*

The sign of *a* determines the shape of the graph.

- If $a > 0$, the graph of $f(x)$ lies in the first and third quadrants.

For $a > 1$, the graph of $f(x)$ will be further away from the axes than $y = \frac{1}{x}$

For $0 < a < 1$, as a tends to 0, the graph moves closer to the axes than $y = \frac{1}{x}$

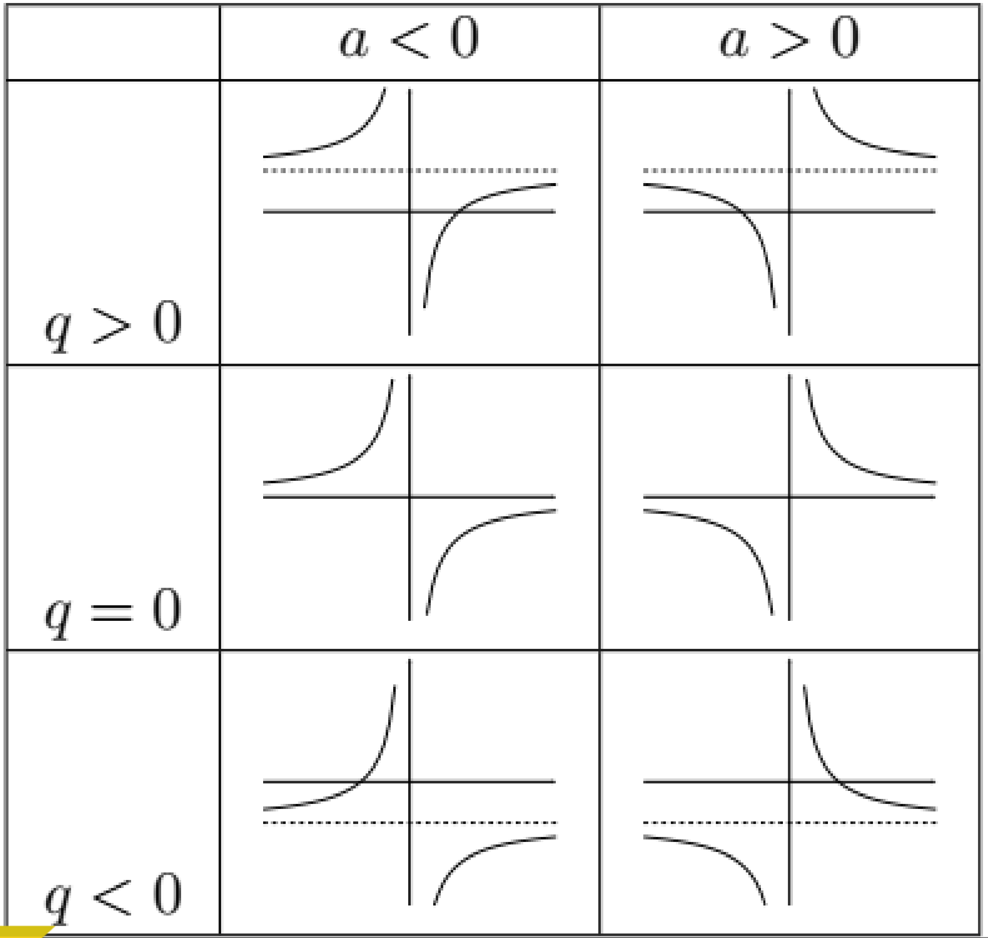
- If $a < 0$, the graph of $f(x)$ lies in the second and fourth quadrants.

For $a < -1$, the graph of $f(x)$ will be further away from the axes than $y = -\frac{1}{x}$

For $-1 < a < 0$, as a tends to 0, the graph moves closer to the axes than $y = -\frac{1}{x}$

FUNCTIONS

The effects of a and q on a hyperbola



FUNCTIONS

Domain and range

For $y = \frac{a}{x} + q$, the function is undefined for $x = 0$. The domain is therefore $\{x : x \in \mathbb{R}, x \neq 0\}$.

We see that $y = \frac{a}{x} + q$ can be rewritten as:

$$\begin{aligned} y &= \frac{a}{x} + q \\ y - q &= \frac{a}{x} \\ \text{If } x \neq 0 \text{ then: } (y - q)x &= a \\ x &= \frac{a}{y - q} \end{aligned}$$

This shows that the function is undefined only at $y = q$.

Therefore the range is $\{f(x) : f(x) \in \mathbb{R}, f(x) \neq q\}$

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FUNCTIONS

Intercepts

The y-intercept:

Every point on the y-axis has an x-coordinate of 0, therefore to calculate the y-intercept let $x = 0$.

The x-intercept:

Every point on the x-axis has a y-coordinate of 0, therefore to calculate the x-intercept, let $y = 0$.

FUNCTIONS

Asymptotes

There are two asymptotes for functions of the form $y = \dfrac{a}{x} + q$.

The horizontal asymptote is the line $y = q$ and the vertical asymptote is always the y-axis, the line $x = 0$.

Axes of symmetry

There are two lines about which a hyperbola is symmetrical: $y = x + q$ and $y = -x + q$.

FUNCTIONS

In order to sketch graphs of functions of the form, $y = f(x) = \frac{a}{x} + q$, we need to determine four characteristics:

1. sign of a
2. y-intercept
3. x-intercept
4. asymptotes

EXERCISE: FUNCTIONS

Sketch the graph of $y = \frac{-4}{x} + 7$

Step 1: Examine the standard form of the equation
We see that $a < 0$ therefore the graph lies in the second and fourth quadrants.

Step 2: Calculate the intercepts
For the *y*-intercept, let $x = 0$:

$$y = \frac{-4}{0} + 7$$

This is undefined, therefore there is no *y*-intercept.

FUNCTIONS

For the *x*-intercept, let *y* = 0:

$$0 = \frac{-4}{x} + 7$$

$$-7 = \frac{-4}{x}$$

$$x = \frac{-4}{-7} = \frac{4}{7}$$

This gives the point $(\frac{4}{7}\,;0)$

FUNCTIONS

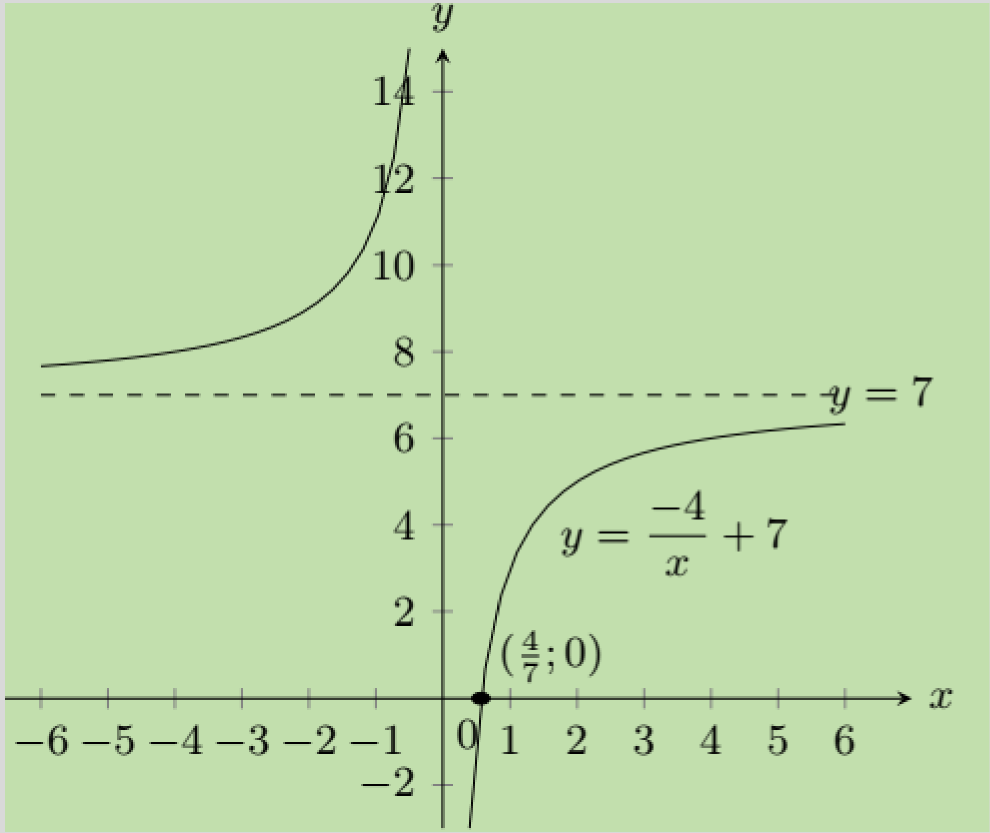
- **Step 3: Determine the asymptotes**
  The horizontal asymptote is the line $y = 7$. The vertical asymptote is the line $x = 0$.
  Step 4: Sketch the graph

Domain: $\{x : x \in \mathbb{R}, x \neq 0\}$

Range: $\{y : y \in \mathbb{R}, y \neq 7\}$

Axis of symmetry: $y = x + 7$ and $y = -x + 7$

FUNCTIONS



EXAMPLE: FUNCTIONS

Consider the function $f(x) = \frac{-4}{x} + 1$. Determine

a) the equations of the asymptotes

b) the coordinates of the x–intercepts.

c) Sketch the graph.

d) Write down the domain and range.

e) If the graph of f is reflected by the line having the equation

y = –x + c, the new graph coincides with the graph of f(x). Determine the value of c.

SOLUTIONS: FUNCTIONS

a) The horizontal asymptote is y = 1 since the graph moved 1 units up and the vertical asymptote is x = 0 since the denominator cannot equal to zero.

b) For x–intercepts let y = 0.

$$y = \frac{-4}{x} + 1$$

$$0 = \frac{-4}{x} + 1$$

$$-1 = \frac{-4}{x}$$

$$x = 4$$

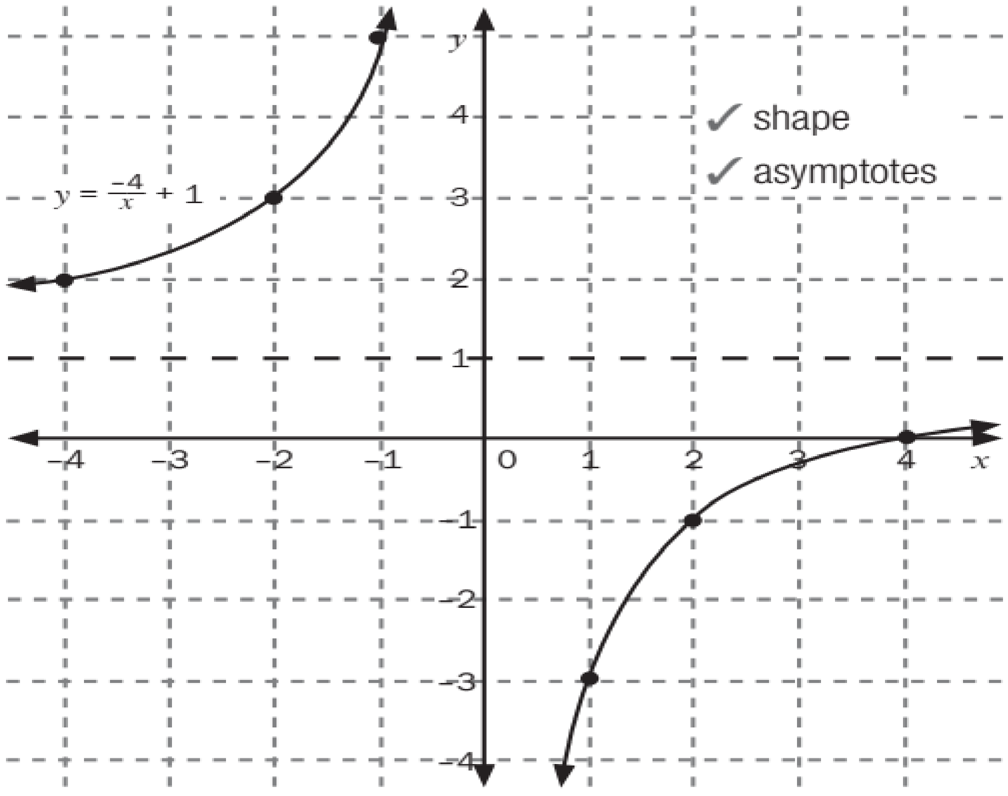
So the intercept is (4; 0)

FUNCTIONS

c)Sketch

of the graph

x	–4	–2	–1	0	1	2	4
y	2	2	5	undefined	–3	–1	0



FUNCTIONS

Domain: $x \in \mathbb{R}; y \neq 0$ ✓

Range: $y \in \mathbb{R}; y \neq 1$ ✓

The asymptotes are

$x = 0$ and $y = 1$

$y = -x + c$

$1 = -(0) + c$

$1 = c$

lines are $y = -x + 1$ and $y = x + 1$ ✓