BELGIUM CAMPUS iTversity

It's the way we're wired

GRADE 12 MATHS

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FUNCTIONS

Functions of the form $y = mx + c$ are called straight line functions. In the equation, $y = mx + c$, m and c are constants and have different effects on the graph of the function.

m is the gradient

As m increases, the gradient of the graph increases.
If $m > 0$ then the graph increases from left to right (slopes upwards).
If $m < 0$ then the graph increases from right to left (slopes downwards)

c is the y intercept

If $c > 0$ the graph shifts vertically upwards.
If $c < 0$ the graph shifts vertically downwards.

FUNCTIONS

Domain and range

The domain is $\{x : x \in \mathbb{R}\}$ because there is no value of x for which $f(x)$ is undefined.

The range of $f(x) = mx + c$ is also $\{f(x) : f(x) \in \mathbb{R}\}$ because $f(x)$ can take on any real value.

FUNCTIONS

Intercepts

The y-intercept:

Every point on the y-axis has an x-coordinate of 0. Therefore to calculate the y-intercept, let $x = 0$

For example, the y-intercept of $g(x) = x - 1$ is given by setting $x = 0$:

$$\begin{aligned} g(x) &= x - 1 \\ g(0) &= 0 - 1 \\ &= -1 \end{aligned}$$

This gives the point $(0; -1)$.

The x-intercept:

Every point on the x-axis has a y-coordinate of 0. Therefore to calculate the x-intercept, let $y = 0$.

For example, the x-intercept of $g(x) = x - 1$ is given by setting $y = 0$:

$$\begin{aligned} g(x) &= x - 1 \\ 0 &= x - 1 \\ \therefore x &= 1 \end{aligned}$$

This gives the point $(1; 0)$.

FUNCTIONS

In order to sketch graphs of the form, $f(x) = mx + c$, we need to determine three characteristics:

1. sign of m
2. y-intercept
3. x-intercept

EXAMPLE: FUNCTIONS

Sketch the graph of $g(x) = x - 1$ using the dual intercept method

<u>Step 1: Examine the standard form of the equation</u>

In this example, $m = 1$ in other words $m > 0$. This means the graph increases as x increases

<u>Step 2: Calculate the intercepts</u>

For the *y*-intercept, let *x* = 0; therefore *g*(0) = -1.

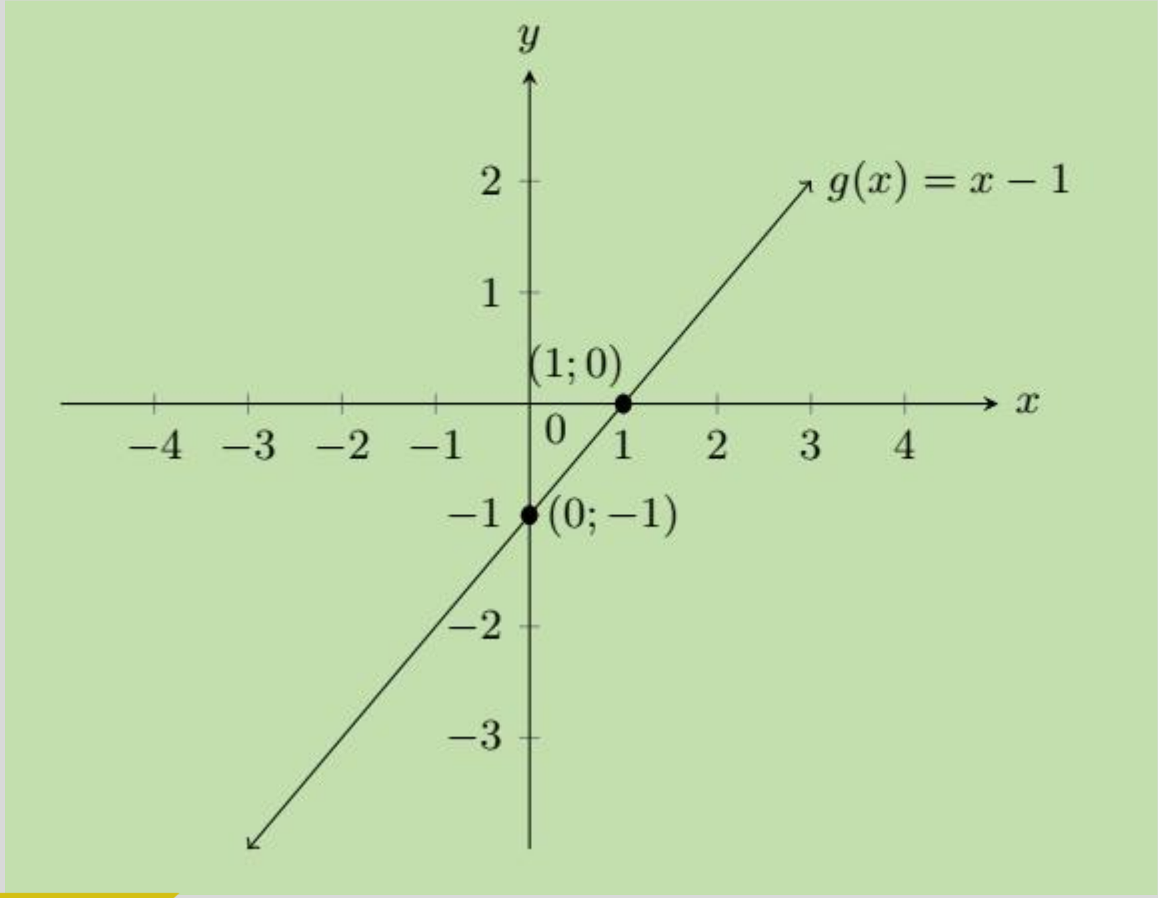
This gives the point (0; -1).

For the *x*-intercept, let *y* = 0; therefore *x* = 1.

This gives the point (1; 0).

EXAMPLE: FUNCTIONS

Step 3: Plot the points and draw the graph



EXAMPLE: FUNCTIONS

Sketch the graph of $p(x) = \frac{1}{2}x - 3$ using the gradient-intercept method.

Step 1: Use the intercept

c = -3, which gives the point (0; -3).

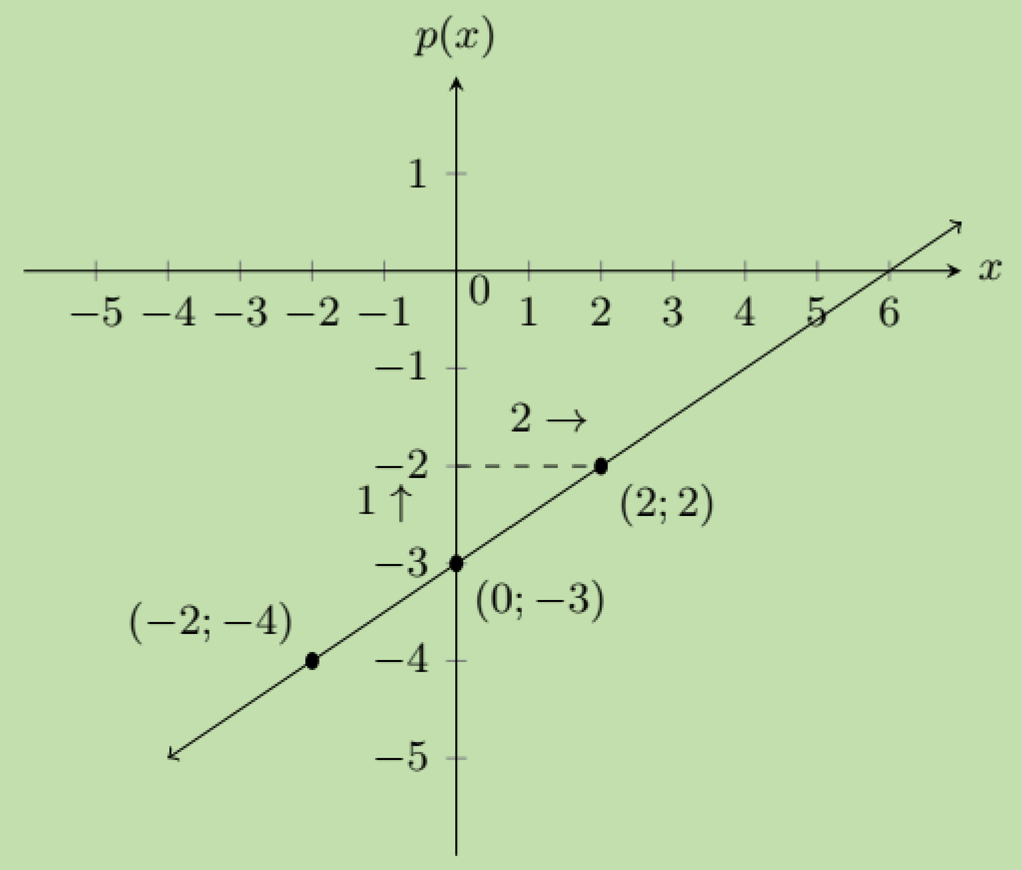
Step 2: Use the gradient

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{1}{2}$$

Start at (0; -3). Move 1 unit up and 2 units to the right. This gives the second point (2; -2).

EXAMPLE: FUNCTIONS

Step 3: Plot the points and draw the graph



EXERCISE: FUNCTIONS

1. Determine the x-intercept and the y-intercept of the following equations.
   a) $y = x - 1$ b) $y = x + 2$ c) $y = x - 3$
2. Write the following in standard form ($y = mx + c$):
   a) $2y + 3x = 1$ b) $3x - y = 5$
   c) $3y - 4 = x$ d) $y + 2x - 3 = 1$

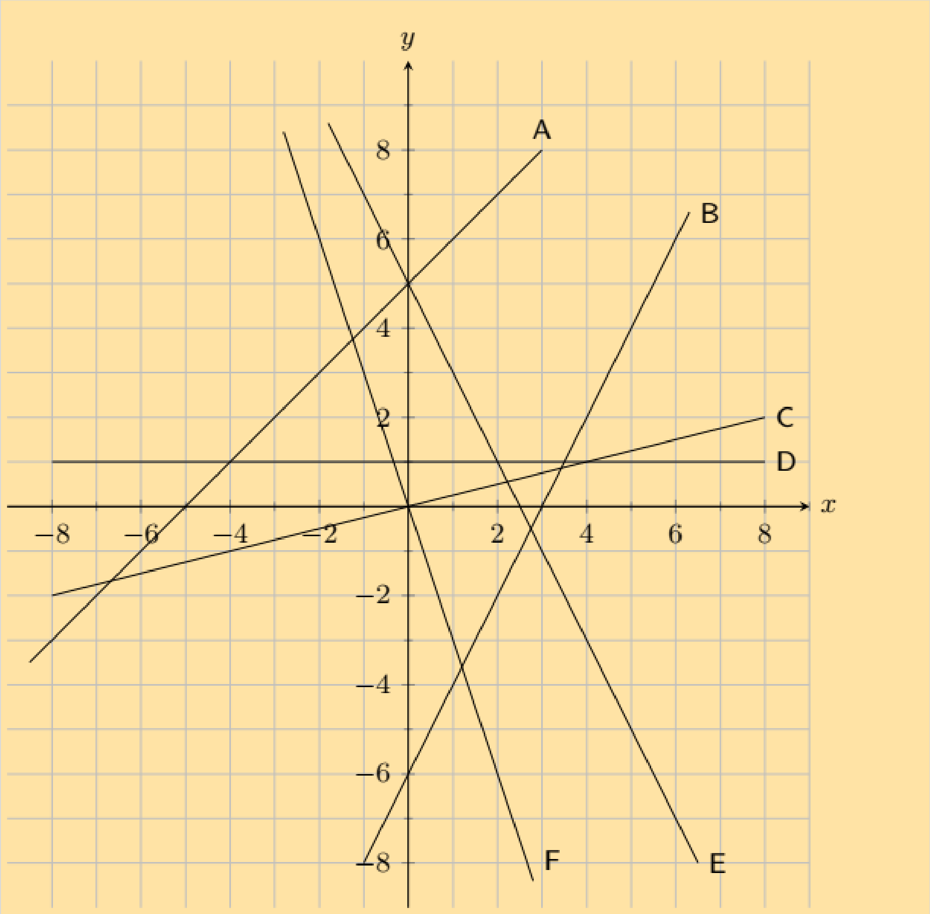
EXERCISE: FUNCTIONS

3. Look at the graphs below. Each graph is labelled with a letter. In the questions that follow, match any given equation with the label of a corresponding graph.

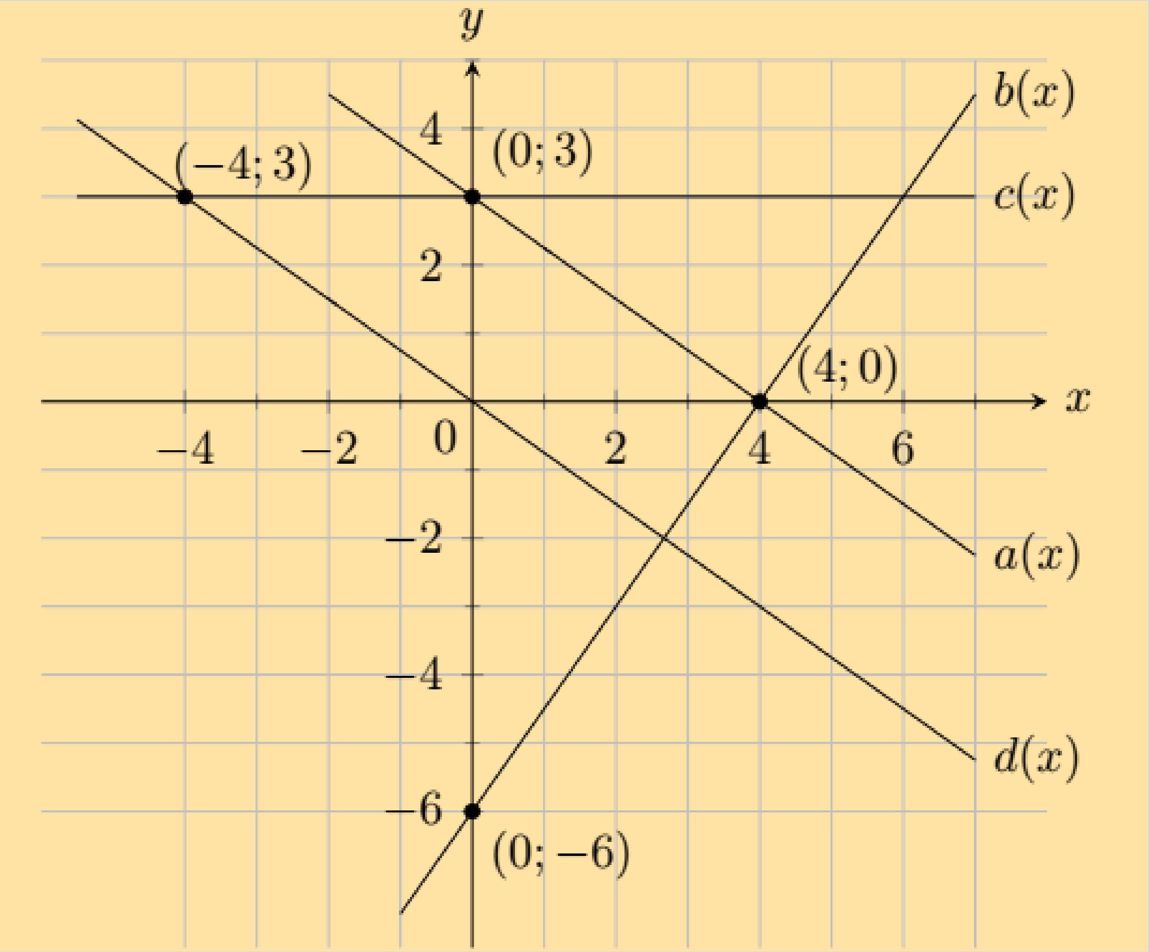
a) $y = 5 - 2x$ b) $x + 5$ c) $y = 2x - 6$

d) $y = -3x$ e) $y = 1$ f) $y = \frac{1}{2}x$

FUNCTIONS



FOR THE FUNCTIONS IN THE DIAGRAM BELOW, GIVE THE EQUATION OF EACH LINE:



SOLUTIONS: FUNCTIONS

1. a) x-intercept = 1 and y-intercept = -1
   b) x-intercept = -2 and y-intercept = 2
   c) x-intercept = 3 and y-intercept = -3
2. a) $y = -\frac{3}{2}x + \frac{1}{2}$

b) $y = -3x + 5$

c) $y = \frac{1}{3}x + \frac{4}{3}$

d) $y = -2x + 4$

SOLUTIONS: FUNCTIONS

3.

a) E
b) A
c) B
d) F
e) D
f) C

4.

$a)\quad a(x) = -\frac{3}{4}x + 3$

b) $b(x) = \frac{3}{2}x - 6$

c) $c(x) = 3$

d) $d(x) = -\frac{3}{4}x$

FUNCTIONS: INVERSE

The inverse of a function takes the y-values (range) of the function to the corresponding x-values (domain) and vice versa. Therefore the x and y values are interchanged.

The function is reflected along the line y = x to form the inverse.

The notation for the inverse of a function is f^{-1}.

FUNCTIONS: INVERSE

Given $f(x) = 2x + 6$. Determine $f^{-1}(x)$. Sketch the graphs of $f(x)$, $f^{-1}(x)$ and $y = x$ on the same set of axis.

<u>Step 1: Swap the *x* and *y*</u>

$y = 2x + 6$ becomes $x = 2y + 6$

<u>Step 2: Make y the subject of the formula</u>

$$2y = x - 6$$

$$y = \frac{1}{2}(x - 6)$$

$$y = \frac{1}{2}x - 3$$

$$\therefore f^{-1} = \frac{1}{2}x - 3$$

FUNCTIONS: INVERSE

Every point on the function has the same coordinates as the corresponding point on the inverse function, *except that they are swapped around*. For example: (–3; 0) on the function is reflected to become (0; –3) on the inverse function.