BELGIUM CAMPUS
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GRADE 12 MATHS

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FUNCTIONS

Functions of the general form $y = ax^2 + bx + c$ are called parabolic functions. The constants a, b and c have different effects on the parabola

The effect of *a*

The sign of *a* determines the shape of the graph.

• For *a* > 0, the graph of *f*(*x*) is a “smile”. The graph of *f*(*x*) is stretched vertically upwards; as *a* gets larger, the graph gets narrower.

For 0 < *a* < 1, as *a* gets closer to 0, the graph of *f*(*x*) gets wider.

• For *a* < 0, the graph of *f*(*x*) is a “frown”. The graph of *f*(*x*) is stretched vertically downwards; as *a* gets smaller, the graph gets narrower.

For -1 < *a* < 0, as *a* gets closer to 0, the graph of *f*(*x*) gets wider

FUNCTIONS

The effect of b:

The x-value of the turning point is determined by $x = -\frac{b}{2a}$. This is the axis of symmetry

The effect of c:

c gives us the y intercept i.e. (0;c)

EXAMPLE: FUNCTIONS

Sketch the graph of $y = \frac{1}{2}x^2 - 4x + \frac{7}{2}$
Determine the intercepts, turning point and the axis of symmetry.
Give the domain and range of the function.

We notice that $a = \frac{1}{2}$; $b = -4$ $c = \frac{7}{2}$

Step 1: Examine the equation of the form $y = ax^2 + bx + c$

We notice that $a > 0$, therefore the graph is a "smile" and has a minimum turning point.

EXAMPLE: FUNCTIONS

Step 2: Determine the turning point and the axis of symmetry

$$x = -\frac{b}{2a}$$

$$x = -\frac{-4}{2\left(\frac{1}{2}\right)} = 4$$

Therefore the axis of symmetry is *x* = 4.
Substitute *x* = 4 into the original equation to obtain the corresponding *y*-value.

$$y = \frac{1}{2}(4)^2 - 4(4) + \frac{7}{2} = -4\frac{1}{2}$$

This gives the point $\left(4; -4\frac{1}{2}\right)$ as the turning point

EXAMPLE: FUNCTIONS

Step 3: Determine the *y*-intercept

The *y*-intercept is obtained by letting *x* = 0: $y = \frac{7}{2}$

This gives the point $\left(0; \frac{7}{2}\right)$ as the y-intercept

Step 4: Determine the *x*-intercepts

The *x*-intercepts are obtained by letting *y* = 0:

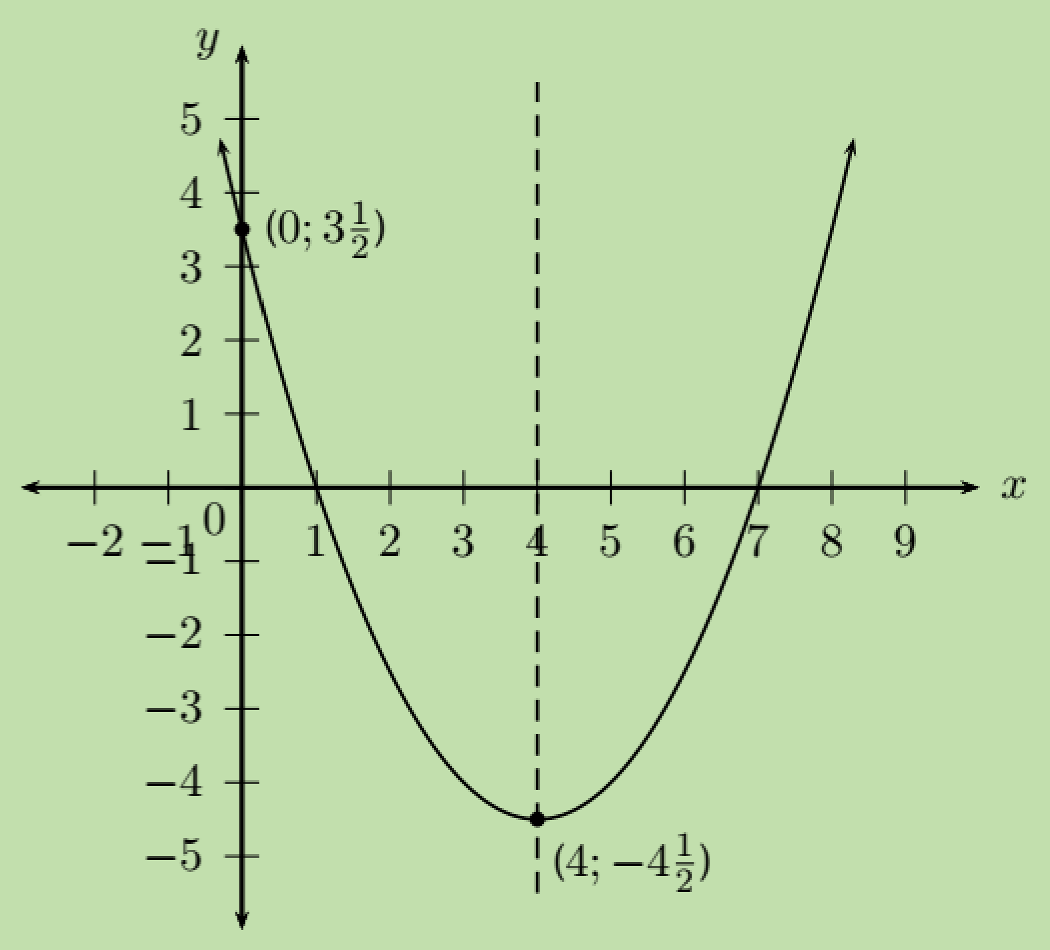
$$\begin{aligned} 0 &= \frac{1}{2}x^2 - 4x + \frac{7}{2} \\ &= x^2 - 8x + 7 \\ &= (x-7)(x-1) \end{aligned}$$

Either x = 7 or 1

This gives the points (1; 0) and (7; 0).

EXAMPLE: FUNCTIONS

- Step 5: Plot the points and sketch the graph



EXAMPLE: FUNCTIONS

<u>Step 6: State the domain and range</u>

- Domain: {x: x ∈ R}
- Range: {y: y$\geq -4\frac{1}{2}, y \in R$}

EXERCISE: FUNCTIONS

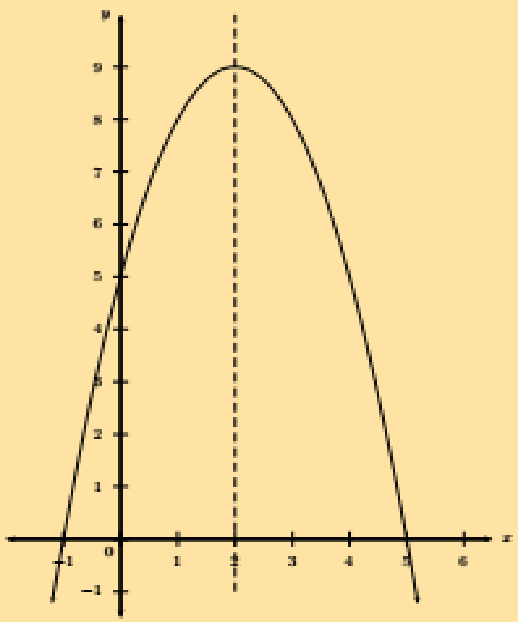
Sketch graphs of the following functions and determine:

- intercepts
- turning point
- axes of symmetry
- domain and range

a) $y = -x^2 + 4x + 5$
b) $y = 2(x+1)^2$
c) $y = 3x^2 - 2(x+2)$
d) $y = 3(x-2)^2 + 1$

SOLUTIONS: FUNCTIONS

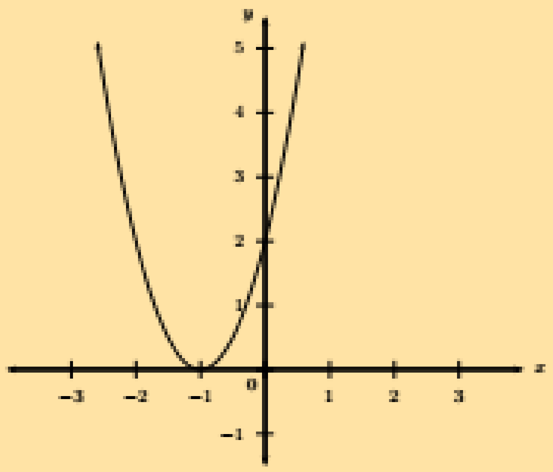
a)



Intercepts: $(-1;0)$, $(5;0)$, $(0;5)$
Turning point: $(2;9)$
Axes of symmetry: $x = 2$
Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y \leq 9, y \in \mathbb{R}\}$

SOLUTIONS: FUNCTIONS

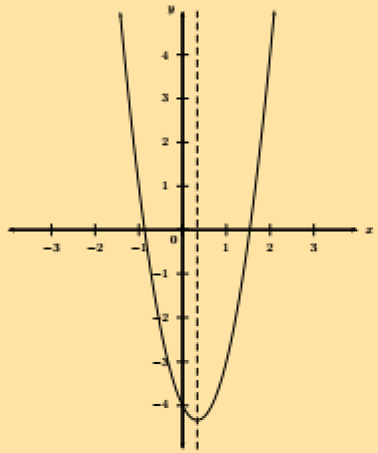
b)



Intercepts: $(-1;0),(0;2)$
Turning point: $(-1;0)$
Axes of symmetry: $x=-1$
Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y \geq 0, y \in \mathbb{R}\}$

SOLUTIONS: FUNCTIONS

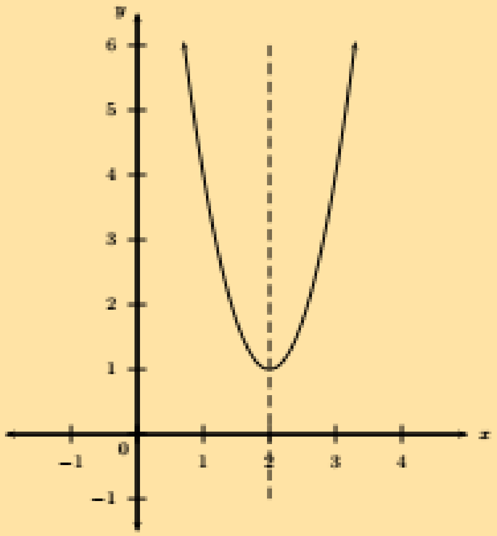
c)



Intercepts: $(-0{,}87; 0), (1{,}54; 0), (0; -4)$
Turning point: $(0{,}33; -4{,}33)$
Axes of symmetry: $x = -0{,}33$
Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y \geq 4{,}33, y \in \mathbb{R}\}$

SOLUTIONS: FUNCTIONS

d)

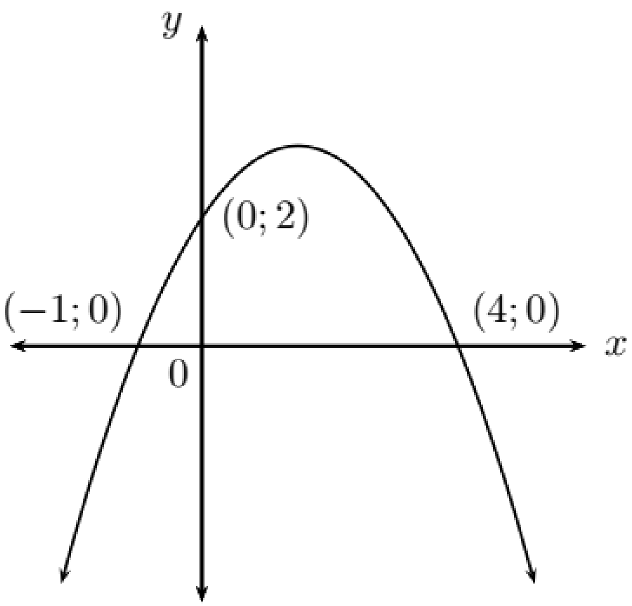


Intercepts: $(0; 13)$ Turning point: $(2; 1)$
Axes of symmetry: $x = 2$
Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y \geq 1, y \in \mathbb{R}\}$

FUNCTIONS: FINDING THE EQUATION OF A PARABOLA

If the intercepts are given, use $y = a(x - x_1)(x - x_2)$.

Example:



x-intercepts: $(-1; 0)$ and $(4; 0)$

$$\begin{aligned} y &= a(x - x_1)(x - x_2) \\ &= a(x+1)(x-4) \\ &= ax^2 - 3ax - 4a \end{aligned}$$

y-intercept: $(0; 2)$

$$-4a = 2$$

$$a = -\frac{1}{2}$$

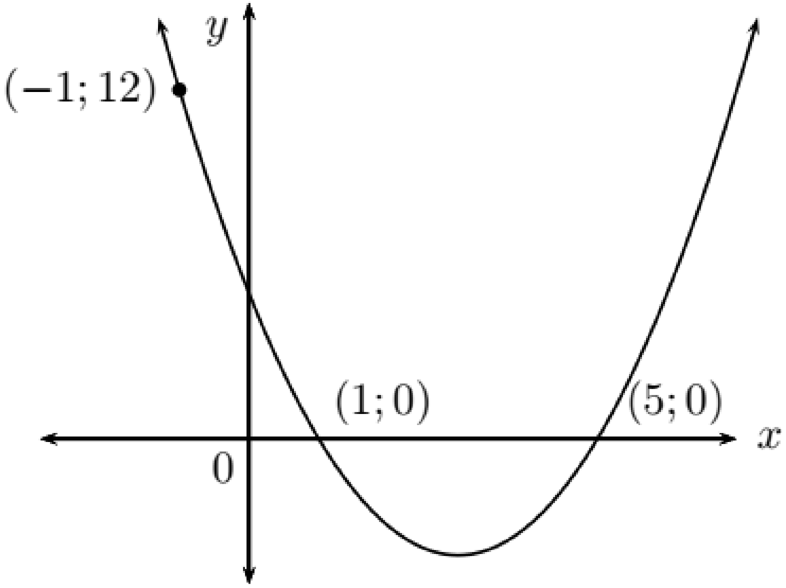
Equation of the parabola:

$$\begin{aligned} y &= ax^2 - 3ax - 4a \\ &= -\frac{1}{2}x^2 - 3\left(-\frac{1}{2}\right)x - 4\left(-\frac{1}{2}\right) \\ &= -\frac{1}{2}x^2 + \frac{3}{2}x + 2 \end{aligned}$$

FUNCTIONS: FINDING THE EQUATION OF A PARABOLA

If the x-intercepts and another point are given, use $y = a(x - x_1)(x - x_2)$.

Example:



x-intercepts: $(1; 0)$ and $(5; 0)$

$$\begin{aligned} y &= a(x - x_1)(x - x_2) \\ &= a(x - 1)(x - 5) \\ &= ax^2 - 6ax + 5a \end{aligned}$$

Substitute the point: $(-1; 12)$

$$\begin{aligned} 12 &= a(-1)^2 - 6a(-1) + 5a \\ 12 &= a + 6a + 5a \\ 12 &= 12a \\ 1 &= a \end{aligned}$$

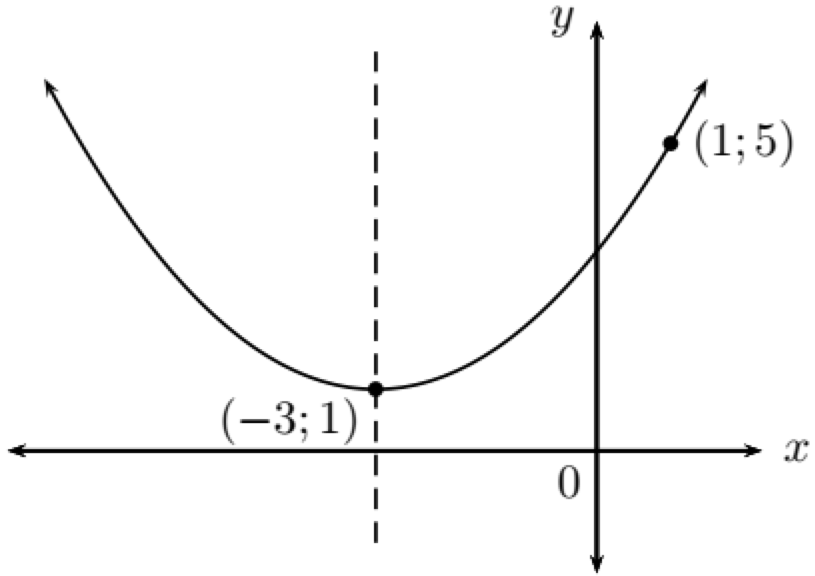
Equation of the parabola:

$$\begin{aligned} y &= ax^2 - 6ax + 5a \\ &= x^2 - 6x + 5 \end{aligned}$$

FUNCTIONS: FINDING THE EQUATION OF A PARABOLA

If the turning point and another point are given, use $y = a(x+p)^2 + q$.

Example:



Turning point: $(-3; 1)$

$$\begin{aligned} y &= a(x+p)^2 + q \\ &= a(x+3)^2 + 1 \\ &= ax^2 + 6ax + 9a + 1 \end{aligned}$$

Substitute the point: $(1; 5)$

$$5 = a(1)^2 + 6a(1) + 9a + 1$$

$$4 = 16a$$

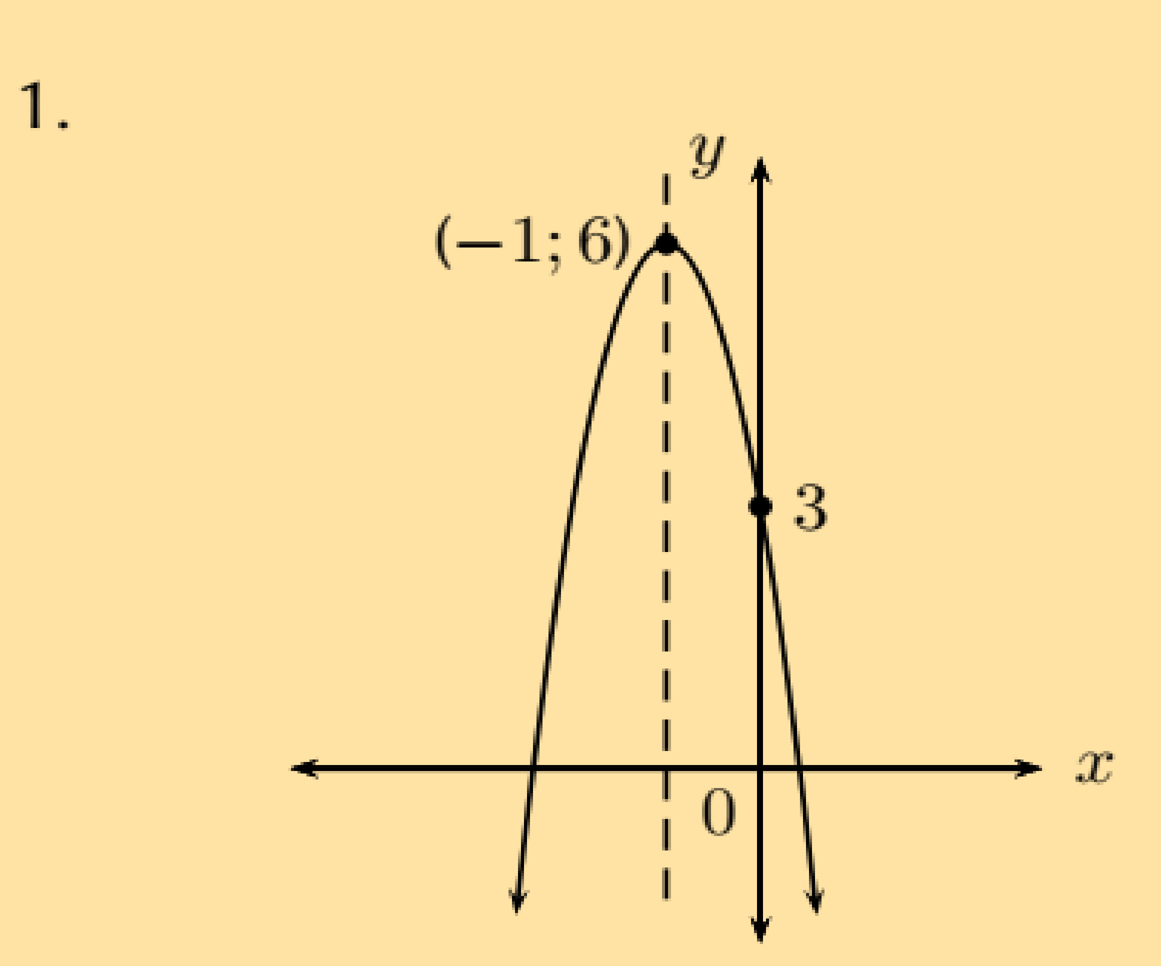
$$\frac{1}{4} = a$$

Equation of the parabola:

$y = \frac{1}{4}(x+3)^2 + 1$

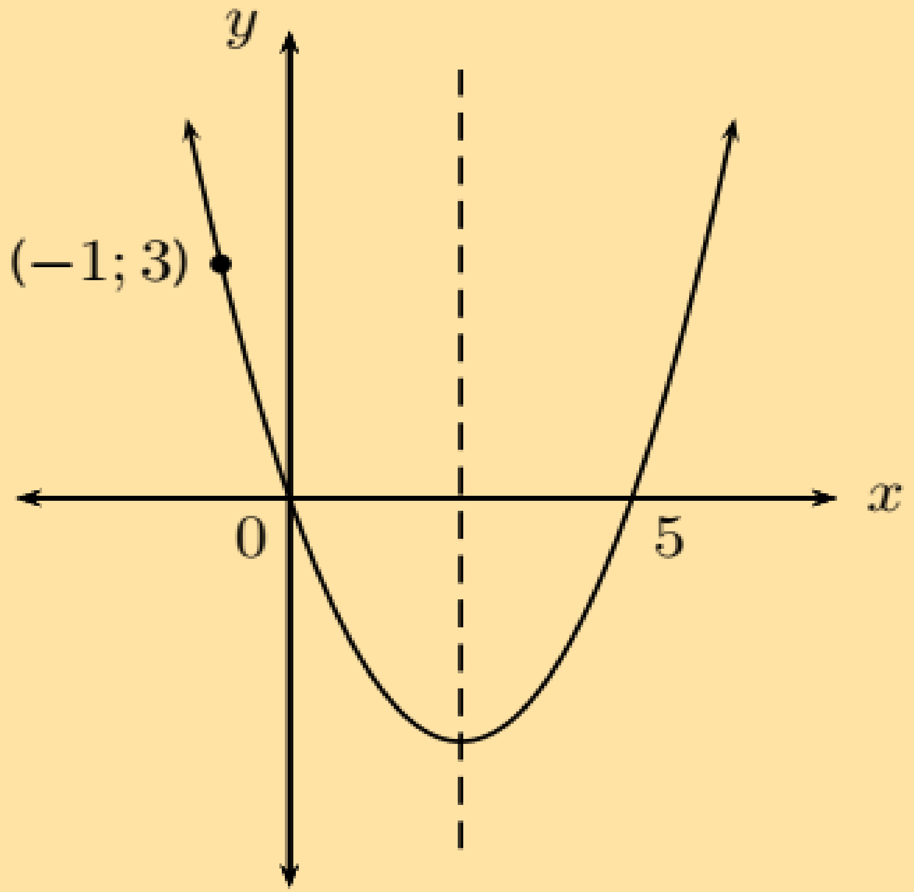
EXERCISE: FUNCTIONS

Determine the equations of the following graphs



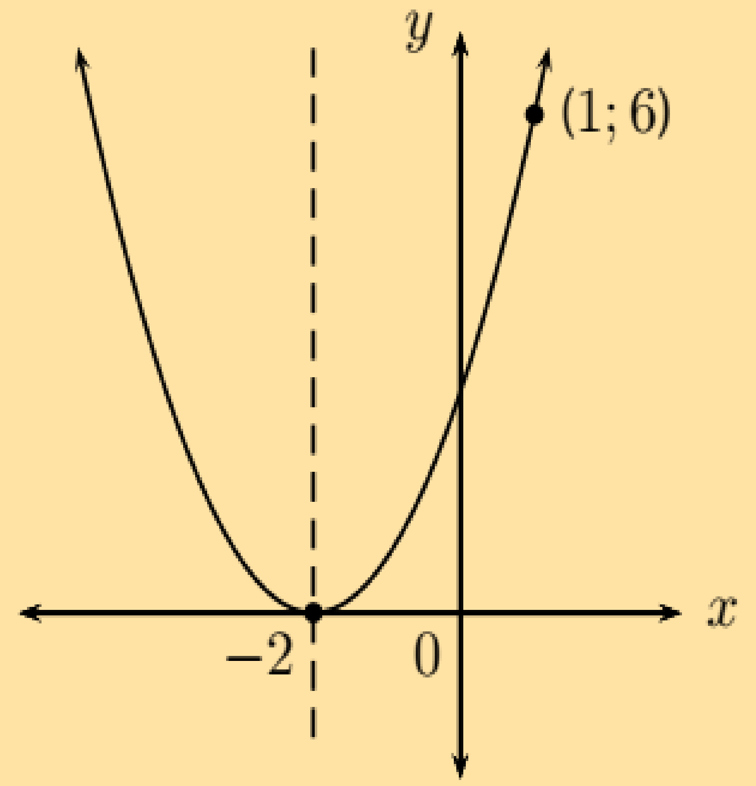
EXERCISE: FUNCTIONS

2.

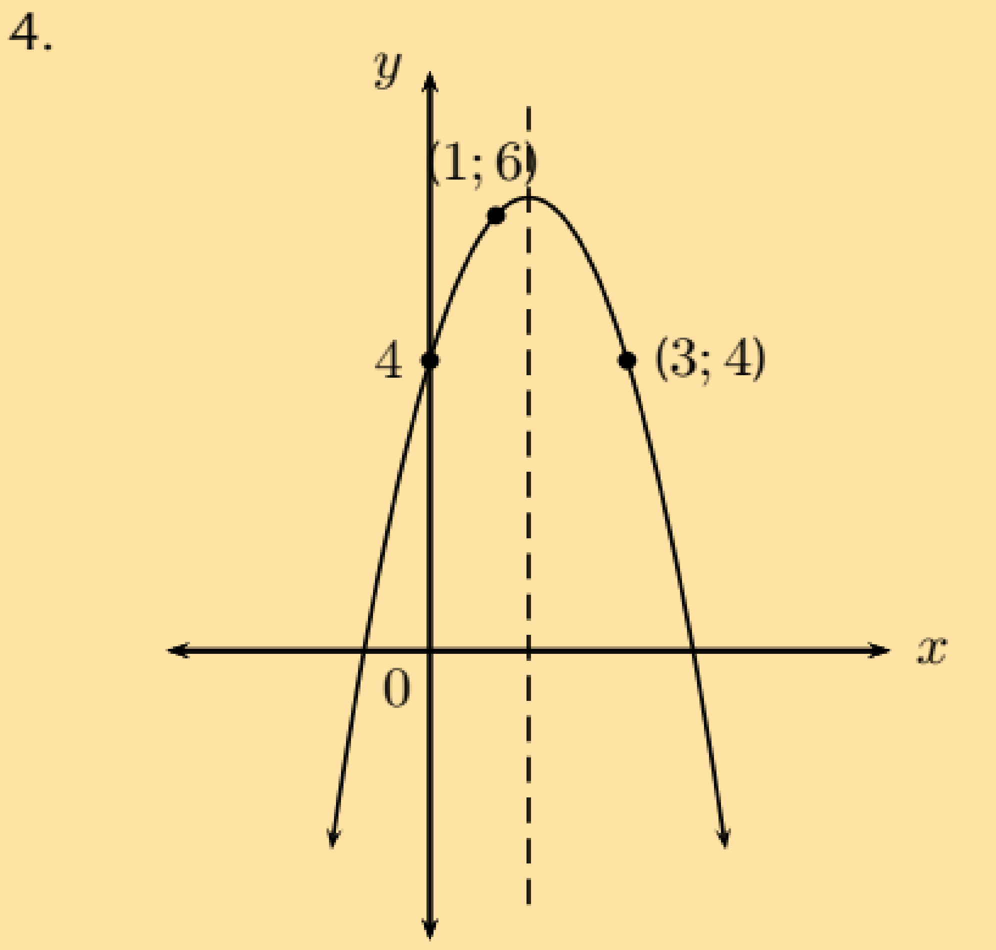


EXERCISE: FUNCTIONS

3.



EXERCISE: FUNCTIONS



SOLUTIONS: FUNCTIONS

1. $y = -3(x+1)^2 + 6$ *or* $y = -3x^2 - 6x + 3$
2. $y = \frac{1}{2}x^2 - \frac{5}{2}x$
3. $y = \frac{2}{3}(x+2)^2$
4. $y = -x^2 + 3x + 4$

FUNCTIONS: INVERSE

Given $f(x) = 2x^2$. Determine the inverse of *f*(*x*). Sketch $f^{-1}(x)$ and *y* = *x* on the same axes as *f*(*x*)

Step 1: Swap the *x* and *y*

$y = 2x^2$ becomes $x = 2y^2$

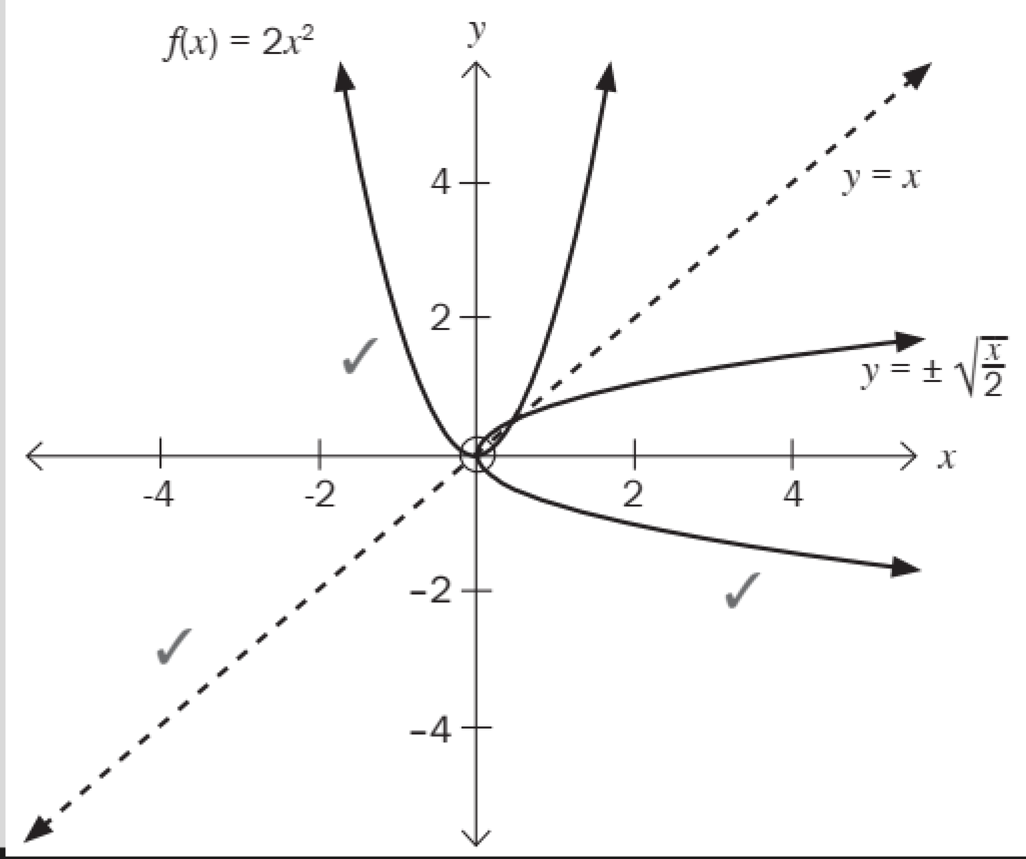
Step 2: Make y the subject of the formula

$$y^2 = \frac{1}{2}x$$

$$y = \pm\sqrt{\frac{1}{2}x}$$

FUNCTIONS: INVERSE

- Not all inverses of functions are also functions. Some inverses of functions are relations. If an inverse is not a function, then we can restrict the **domain** of the **function** in order for the inverse to be a function.



FUNCTIONS: INVERSE

- To check for a function, draw a vertical line. If any vertical line cuts the graph in only one place, the graph is a function. If any vertical line cuts the graph in more than one place, then the graph is not a function.

- To check for a one-to-one function, draw a horizontal line. If any horizontal line cuts the graph in only one place, the graph is a one-to-one function. If any horizontal line cuts the graph in more than one place, then the graph is a many-to-one function.