

BELGIUM CAMPUS
iversity
It's the way we're *wired*



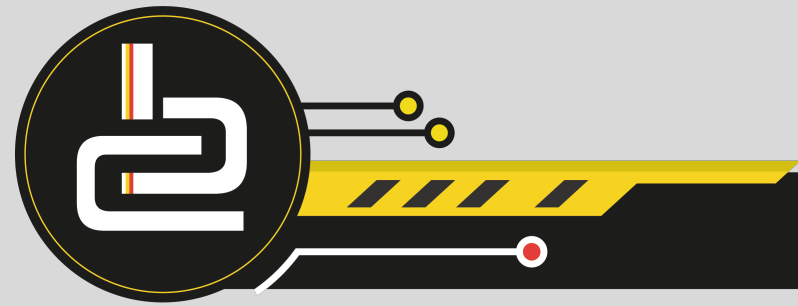
Belgium Campus Spring School



It's the way we're *wired*

GRADE 12 **MATHS**

Charmaine Tavagwisa



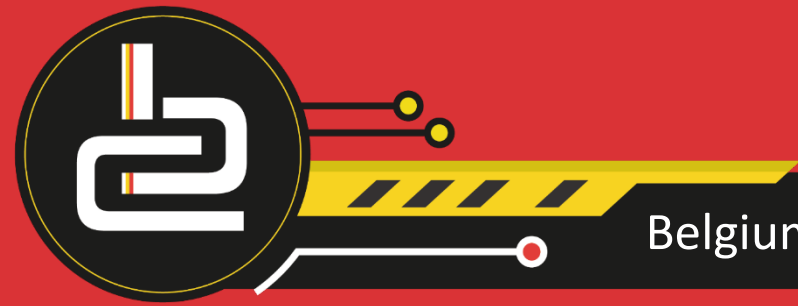


Belgium Campus Spring School

ARE YOU READY?

LESSON OBJECTIVES

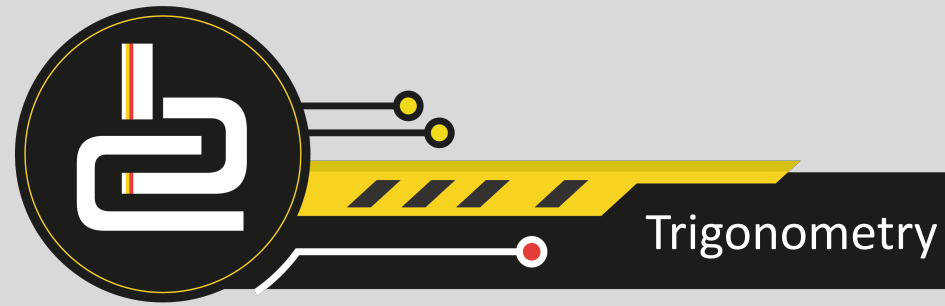
- Trigonometry Ratios
- Special Angles
- CAST diagram
- Reduction Formulas
- Co-Functions
- Area, Sine, Cosine Rule
- Identities



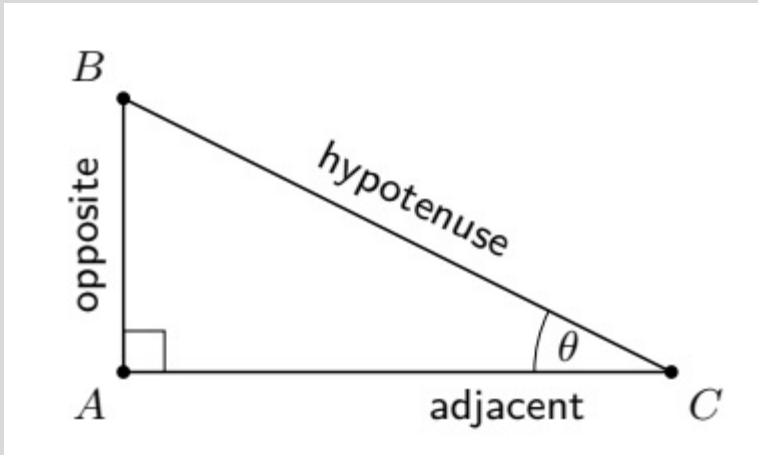
TRIGONOMETRY

TRIGONOMETRY DEALS WITH THE RELATIONSHIP BETWEEN THE ANGLES AND SIDES OF A TRIANGLE.

THERE ARE MANY APPLICATIONS OF TRIGONOMETRY. OF PARTICULAR VALUE IS THE TECHNIQUE OF TRIANGULATION, WHICH IS USED IN ASTRONOMY TO MEASURE THE DISTANCES TO NEARBY STARS, IN GEOGRAPHY TO MEASURE DISTANCES BETWEEN LANDMARKS, AND IN SATELLITE NAVIGATION SYSTEMS.



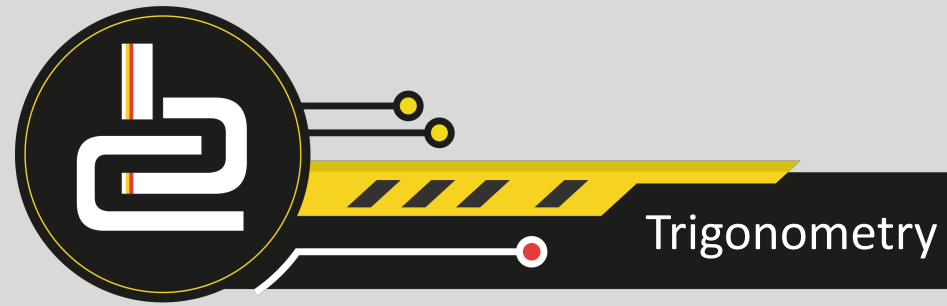
TRIGONOMETRY RATIOS



$$\bullet \sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\bullet \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\bullet \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$



TRIGONOMETRY: RECIPROCAL RATIOS

- $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

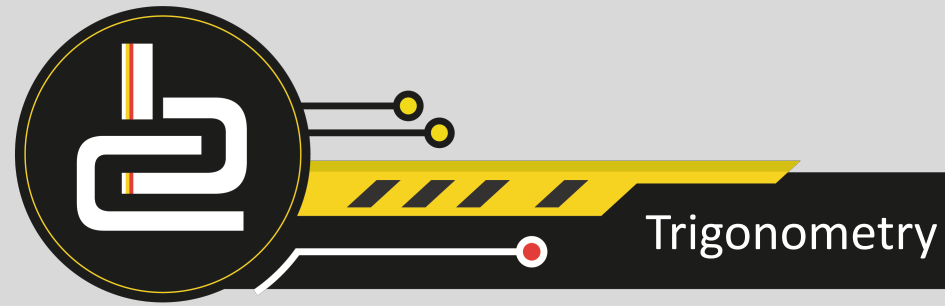
- $\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{opposite}}$

- $\sec \theta = \frac{1}{\cos \theta}$

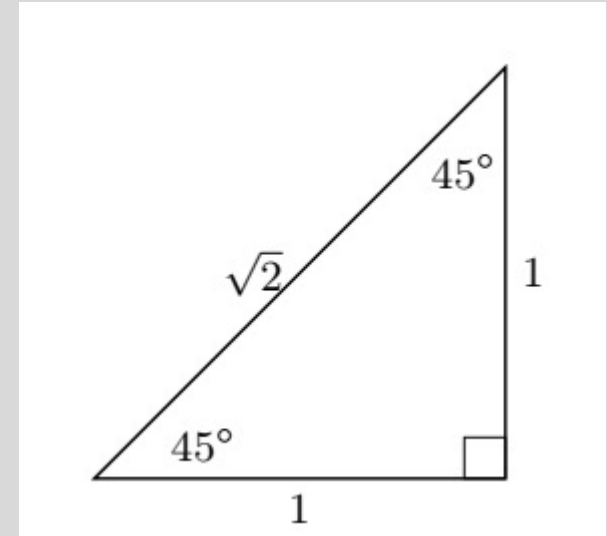
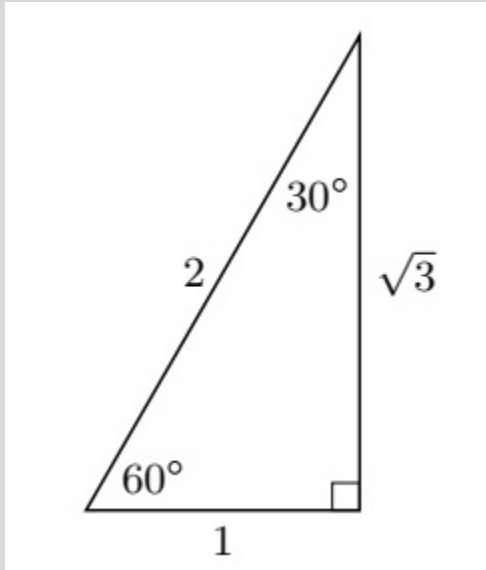
- $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

- $\cot \theta = \frac{1}{\tan \theta}$

- $\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$



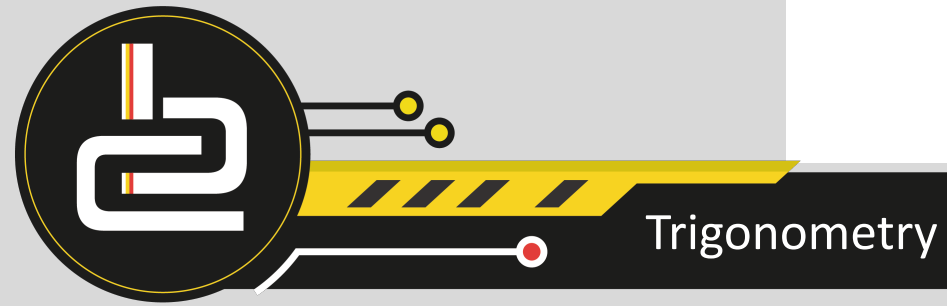
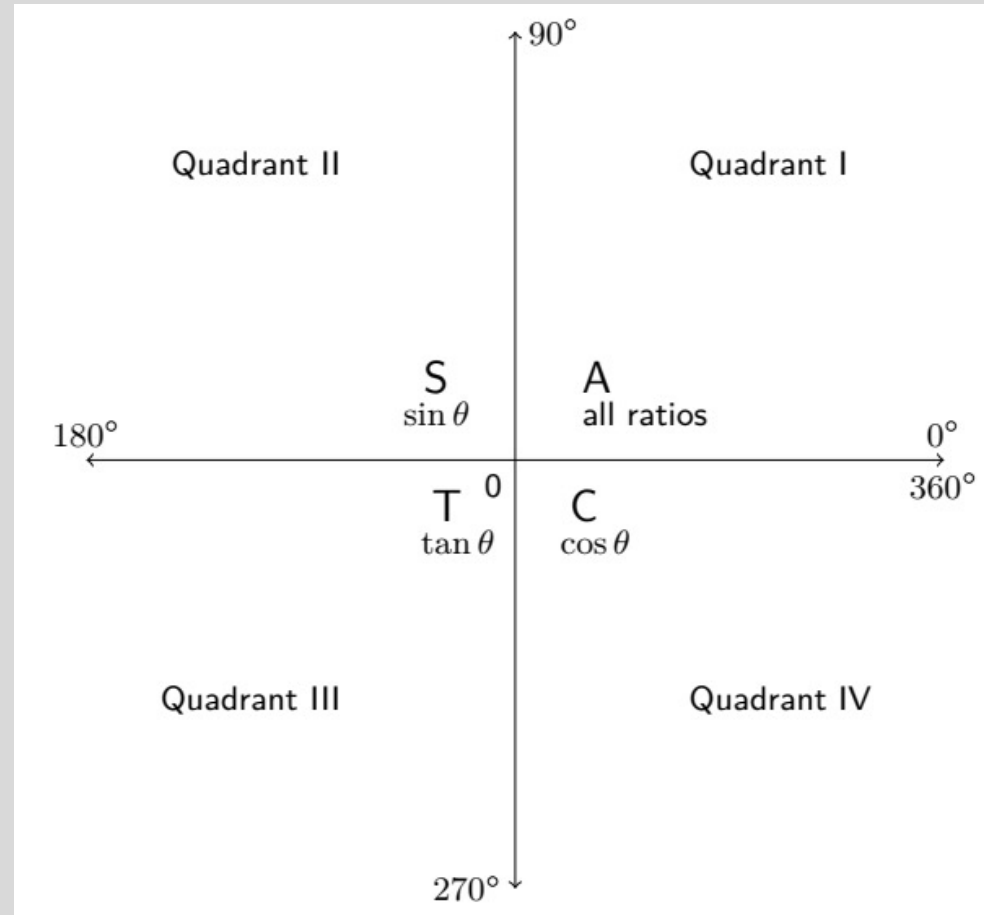
TRIGONOMETRY: SPECIAL ANGLES



θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined



TRIGONOMETRY: CAST



TRIGONOMETRY: REDUCTION FORMULAE & CO-FUNCTIONS

second quadrant ($180^\circ - \theta$) or ($90^\circ + \theta$)	first quadrant (θ) or ($90^\circ - \theta$)
$\sin(180^\circ - \theta) = +\sin \theta$	all trig functions are positive
$\cos(180^\circ - \theta) = -\cos \theta$	$\sin(360^\circ + \theta) = \sin \theta$
$\tan(180^\circ - \theta) = -\tan \theta$	$\cos(360^\circ + \theta) = \cos \theta$
$\sin(90^\circ + \theta) = +\cos \theta$	$\tan(360^\circ + \theta) = \tan \theta$
$\cos(90^\circ + \theta) = -\sin \theta$	$\sin(90^\circ - \theta) = \cos \theta$
	$\cos(90^\circ - \theta) = \sin \theta$
third quadrant ($180^\circ + \theta$)	fourth quadrant ($360^\circ - \theta$)
$\sin(180^\circ + \theta) = -\sin \theta$	$\sin(360^\circ - \theta) = -\sin \theta$
$\cos(180^\circ + \theta) = -\cos \theta$	$\cos(360^\circ - \theta) = +\cos \theta$
$\tan(180^\circ + \theta) = +\tan \theta$	$\tan(360^\circ - \theta) = -\tan \theta$



TRIGONOMETRY

sine rule	area rule	cosine rule
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$\text{area } \triangle ABC = \frac{1}{2}bc \sin A$	$a^2 = b^2 + c^2 - 2bc \cos A$
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$\text{area } \triangle ABC = \frac{1}{2}ac \sin B$	$b^2 = a^2 + c^2 - 2ac \cos B$
	$\text{area } \triangle ABC = \frac{1}{2}ab \sin C$	$c^2 = a^2 + b^2 - 2ab \cos C$



TRIGONOMETRY

How to determine which rule to use:

1. Area rule:

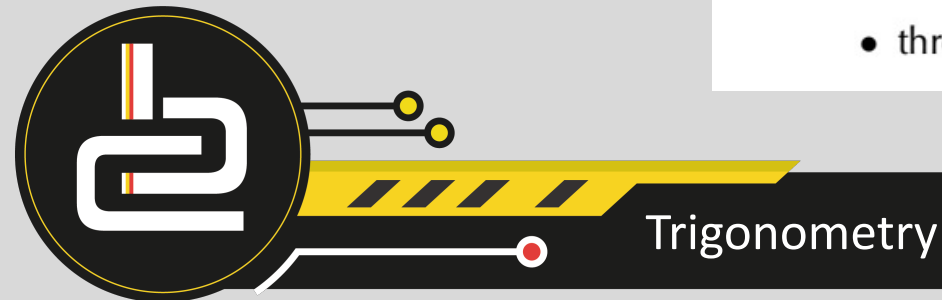
- no perpendicular height is given

2. Sine rule:

- no right angle is given
- two sides and an angle are given (not the included angle)
- two angles and a side are given

3. Cosine rule:

- no right angle is given
- two sides and the included angle are given
- three sides are given



TRIGONOMETRY: IDENTITIES

Pythagorean Identities	Ratio Identities
$\cos^2 \theta + \sin^2 \theta = 1$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\cos^2 \theta = 1 - \sin^2 \theta$	$\frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$
$\sin^2 \theta = 1 - \cos^2 \theta$	



TRIGONOMETRY: IDENTITIES

Compound Angle Identities	Double Angle Identities
$\sin(\theta + \beta) = \sin \theta \cos \beta + \cos \theta \sin \beta$	$\sin(2\theta) = 2 \sin \theta \cos \theta$
$\sin(\theta - \beta) = \sin \theta \cos \beta - \cos \theta \sin \beta$	$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$
$\cos(\theta + \beta) = \cos \theta \cos \beta - \sin \theta \sin \beta$	$\cos(2\theta) = 1 - 2\sin^2 \theta$
$\cos(\theta - \beta) = \cos \theta \cos \beta + \sin \theta \sin \beta$	$\cos(2\theta) = 2\cos^2 \theta - 1$
	$\tan(2\theta) = \frac{\sin 2\theta}{\cos 2\theta}$



TRIGONOMETRY: GENERAL SOLUTION

1.

If $\sin \theta = x$

$$\theta = \sin^{-1} x + k \cdot 360^\circ$$

$$\text{or } \theta = (180^\circ - \sin^{-1} x) + k \cdot 360^\circ$$

2.

If $\cos \theta = x$

$$\theta = \cos^{-1} x + k \cdot 360^\circ$$

$$\text{or } \theta = (360^\circ - \cos^{-1} x) + k \cdot 360^\circ$$

3.

If $\tan \theta = x$

$$\theta = \tan^{-1} x + k \cdot 180^\circ$$

for $k \in \mathbb{Z}$.

